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Mathematics Class 8 — Punjab Board (SNC)

UNIT 8

Expansion and Factorization

Key Concepts · Definitions · Short & Long Questions · MCQs · Diagrams · Exam Tips

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Unit 8: Expansion and Factorization

Unit 8 builds the main tools of algebra. It begins with open and closed sentences and polynomials, then shows how to add, subtract, multiply and divide polynomials. Algebraic identities such as $(a + b)^2 = a^2 + 2ab + b^2$ make expansion quick, and factorization reverses expansion by writing an expression as a product.

The unit also covers exponents (powers), the laws of exponents including zero, negative and fractional indices, and standard form / scientific notation for writing very large and very small numbers.

Learning Objectives

- Distinguish open and closed sentences and identify polynomials, their degree and type.
- Add, subtract, multiply and divide polynomials.
- Use algebraic identities to expand expressions and to calculate values like 103^2 and 101×99.
- Factorize expressions by common factors, grouping, identities and splitting the middle term.
- Apply the laws of exponents, including zero, negative and fractional indices.
- Write numbers in standard form and scientific notation.

Key Concepts

8.1 Sentences and Polynomials

A closed sentence is a statement that is clearly true or false, e.g., $5 + 3 = 8$. An open sentence contains a variable and its truth depends on the value, e.g., $x + 3 = 7$.

A polynomial is an algebraic expression whose terms have variables with whole-number exponents only (0, 1, 2, ...), e.g., $3x^2 – 5x + 2$. Expressions such as $1/x + 2$ or $\sqrt{x} + 1$ are not polynomials.

In $7x^2$, 7 is the coefficient and x is the variable; a term without a variable is the constant term. A polynomial may have one variable ($x^2 + 1$), two variables ($x^2y + xy$) or more.

The degree is the highest exponent (or highest sum of exponents in a term). Degree 1: linear; 2: quadratic; 3: cubic; 4: biquadratic. Example: $x^2y + xy$ has degree 3.

8.2 Operations on Polynomials

Add or subtract by combining like terms: $(3x^2 + 2x – 1) + (x^2 – 5x + 4) = 4x^2 – 3x + 3$. When subtracting, change the sign of every term of the second polynomial.

Multiply each term of one polynomial by each term of the other: $(x + 2)(x + 3) = x^2 + 5x + 6$. The degree of the product is the sum of the degrees.

Divide by long division, as with numbers: $(x^2 + 5x + 6) \div (x + 2) = x + 3$ with remainder 0, so $x + 2$ is a factor. If the remainder is 0, the division is exact.

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8.3 Algebraic Identities

$(a + b)^2 = a^2 + 2ab + b^2$; $(a - b)^2 = a^2 - 2ab + b^2$; $(a + b)(a - b) = a^2 - b^2$.

$(a + b)^3 = a^3 + 3ab(a + b) + b^3$; $(a - b)^3 = a^3 - 3ab(a - b) - b^3$.

Using identities for quick calculation: $103^2 = (100 + 3)^2 = 10000 + 600 + 9 = 10609$; $99^2 = (100 - 1)^2 = 9801$; $101 \times 99 = 100^2 - 1^2 = 9999$.

8.4 Factorization

Factorization writes an expression as a product of its factors; it is the reverse of expansion.

Common factor: $6x^2 + 9x = 3x(2x + 3)$. Grouping: $ax + ay + bx + by = a(x + y) + b(x + y) = (a + b)(x + y)$.

Using identities: $x^2 - 49 = (x + 7)(x - 7)$; $x^2 + 10x + 25 = (x + 5)^2$; $a^2 + 2ab + b^2 - c^2 = (a + b)^2 - c^2 = (a + b + c)(a + b - c)$.

Splitting the middle term: for $x^2 + 7x + 12$ find two numbers with product 12 and sum 7 (3 and 4), so $x^2 + 7x + 12 = (x + 3)(x + 4)$. For $3x^2 + 7xy + 2y^2$, split $7xy$ as $6xy + xy$: $(3x + y)(x + 2y)$.

8.5 Exponents and Their Laws

In a^n, a is the base and n is the exponent (index): $2^5 = 32$. 10^5 is read '10 raised to the power 5'.

Laws: $a^m \times a^n = a^{m+n}$; $a^m \div a^n = a^{m-n}$; $(a^m)^n = a^{mn}$; $(ab)^n = a^n b^n$; $(a/b)^n = a^n/b^n$.

Special indices: $a^0 = 1$ ($a \neq 0$); $a^{-n} = 1/a^n$; a^(1/n) is the nth root of a, so 16^(1/2) = 4 and 27^(2/3) = $(\sqrt[3]{27})^2 = 9$.

8.6 Standard Form and Scientific Notation

Scientific notation writes a number as $a \times 10^n$ where $1 \leq a < 10$ and n is an integer: $4{,}500{,}000 = 4.5 \times 10^6$ and $0.00072 = 7.2 \times 10^{-4}$.

Large numbers have a positive power of 10; small numbers (less than 1) have a negative power. The speed of light, about 300,000,000 m/s, is 3×10^8 m/s.

Important Definitions

Closed sentence
A statement that is definitely true or false.

Open sentence
A sentence with a variable whose truth depends on its value.

Polynomial
An expression whose variables have only whole-number exponents.

Coefficient
The number multiplying a variable in a term.

Degree of a polynomial
The highest exponent (or sum of exponents) of any term.

Quadratic polynomial
A polynomial of degree 2.

Cubic polynomial
A polynomial of degree 3.

Biquadratic polynomial
A polynomial of degree 4.

Identity
An equation true for all values of its variables.

Factorization
Writing an expression as a product of factors.

Base and exponent
In a^n, a is the base and n is the exponent.

Scientific notation
Writing a number as $a \times 10^n$ with $1 \le a < 10$.

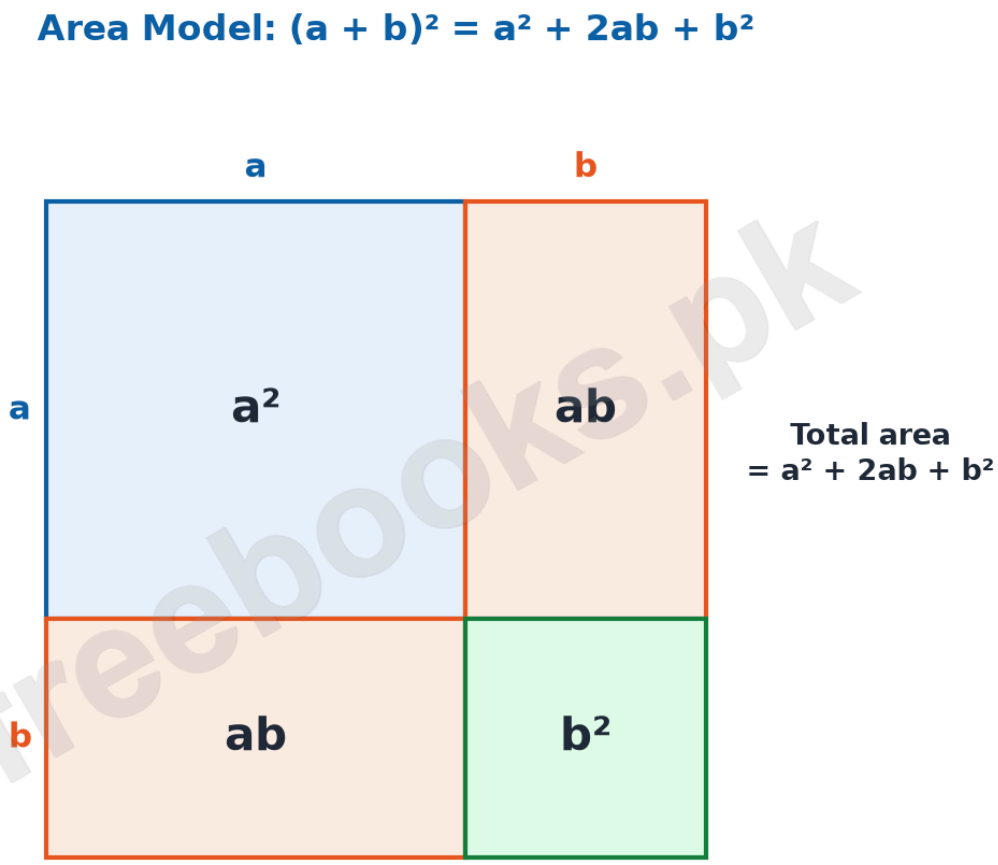
Key Facts at a Glance

Identity / law	Form
$(a + b)^2$	$a^2 + 2ab + b^2$
$(a – b)^2$	$a^2 – 2ab + b^2$
$a^2 – b^2$	$(a + b)(a – b)$
$(a + b)^3$	$a^3 + 3ab(a + b) + b^3$
$(a – b)^3$	$a^3 – 3ab(a – b) – b^3$
$a^m \times a^n$	a^{m+n}
$a^m \div a^n$	a^{m-n}
$(a^m)^n$	a^{mn}
a^0	$1\ (a \neq 0)$
a^{-n}	$1/a^n$

103^2	10609
0.00072	7.2×10^{-4}

Diagrams

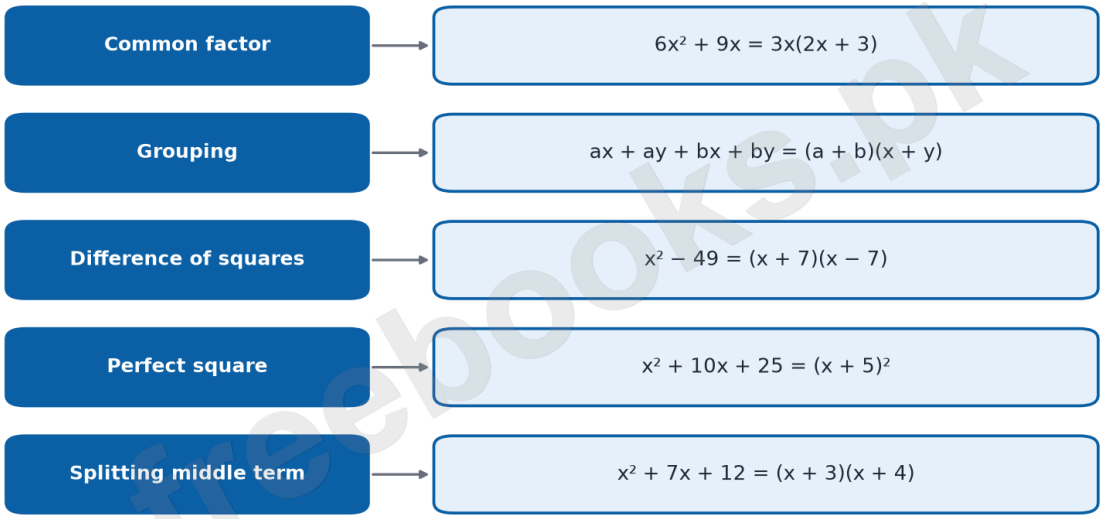
Area model of $(a + b)^2$: A square of side a + b divided into a^2, two ab rectangles and b^2.



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Methods of factorization: A chart of the main factorization methods with an example of each.

Methods of Factorization



Laws of exponents: A chart of the laws of exponents with numerical examples.

Laws of Exponents



Short Questions and Answers

Is x + 5 = 9 an open or closed sentence?
Open, because its truth depends on x.

Is $2x^2 - 3/x$ a polynomial? Why?
No; $3/x = 3x^{-1}$ has a negative exponent.

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Find the degree of $4x^3y + 2xy - 7$.
4 (3 + 1 in the first term).

Subtract $(2a^2 - a + 3)$ from $(5a^2 + 4a - 1)$.
$3a^2 + 5a - 4$.

Multiply $(2x - 1)(x + 4)$.
$2x^2 + 7x - 4$.

Divide $x^2 + 8x + 15$ by $x + 3$.
$x + 5$, remainder 0.

Expand $(2x + 3)^2$.
$4x^2 + 12x + 9$.

Find 98^2 using an identity.
$(100 - 2)^2 = 10000 - 400 + 4 = 9604$.

Find 105×95 using an identity.
$100^2 - 5^2 = 9975$.

Expand $(x + 2)^3$.
$x^3 + 6x^2 + 12x + 8$.

Factorize $9a^2 - 16b^2$.
$(3a + 4b)(3a - 4b)$.

Factorize $x^2 - 5x + 6$.
$(x - 2)(x - 3)$.

Simplify $2^3 \times 2^4 \div 2^5$.
$2^2 = 4$.

Evaluate $5^0 + 2^{-2} + 8\text{^}(1/3)$.
$1 + 1/4 + 2 = 3.25$.

Write 0.0000056 in scientific notation.
5.6×10^{-6}.

Long Questions and Answers

Explain polynomials and their types.

What is a polynomial?

An algebraic expression in which every variable has a whole-number exponent, such as $2x^3 – x + 4$.

How are polynomials named by variables?

In one variable ($x^2 + 1$), two variables ($x^2y – y$) or more ($xyz + 2$).

How are they named by degree?

Linear (1), quadratic (2), cubic (3) and biquadratic (4).

Classify $5x^4 – x^2 + 1$ and $3x – 2$.

The first is biquadratic (degree 4); the second is linear.

Explain the operations on polynomials.

Add $(4x^2 – 3x + 2)$ and $(x^2 + 5x – 7)$.

$5x^2 + 2x – 5$.

Multiply $(x + 3)(x^2 – 2x + 1)$.

$x^3 + x^2 – 5x + 3$.

Divide $x^3 – 1$ by $x – 1$.

$x^2 + x + 1$ with remainder 0.

What is the degree of a product?

The sum of the degrees of the factors: degree 1 × degree 2 gives degree 3.

State and apply the algebraic identities.

Write the square identities.

$(a \pm b)^2 = a^2 \pm 2ab + b^2$ and $(a + b)(a – b) = a^2 – b^2$.

Write the cube identities.

$(a + b)^3 = a^3 + 3ab(a + b) + b^3$ and $(a – b)^3 = a^3 – 3ab(a – b) – b^3$.

Expand $(a – 1)^3$.

$a^3 – 3a(a – 1) – 1 = a^3 – 3a^2 + 3a – 1$.

If $a + b = 7$ and $ab = 10$, find $a^2 + b^2$.

$(a + b)^2 – 2ab = 49 – 20 = 29$.

Explain the methods of factorization with examples.

Taking a common factor

$12x^2y – 18xy^2 = 6xy(2x – 3y)$.

Grouping

$px – py + qx – qy = p(x – y) + q(x – y) = (p + q)(x – y)$.

Using identities

$4x^2 + 12x + 9 = (2x + 3)^2$; $x^2 – 2xy + y^2 – z^2 = (x – y + z)(x – y – z)$.

Splitting the middle term

$2x^2 + 7x + 3$: product 6, sum 7 → 6 and 1: $2x^2 + 6x + x + 3 = (2x + 1)(x + 3)$.

Explain the laws of exponents.

Product and quotient laws

$a^m \times a^n = a^{m+n}$ and $a^m \div a^n = a^{m-n}$; e.g., $3^4 \times 3^2 = 3^6$ and $7^5 \div 7^3 = 49$.

Power laws

$(a^m)^n = a^{mn}$, $(ab)^n = a^n b^n$, $(a/b)^n = a^n/b^n$; e.g., $(3^2)^3 = 729$.

Zero and negative indices

$a^0 = 1$ and $a^{-n} = 1/a^n$; e.g., $(2/3)^{-2} = 9/4$.

Fractional indices

a^(m/n) = $(\sqrt[n]{a})^m$; e.g., 27^(2/3) = 9 and 32^(1/5) = 2.

Explain scientific notation with examples.

What is scientific notation?

Writing a number as $a \times 10^n$ with $1 \le a < 10$ and n an integer.

Write 72,000,000 in scientific notation.

7.2×10^7.

Write 0.000304 in scientific notation.

3.04×10^{-4}.

Write 6.1×10^5 in ordinary form.

610,000.

Multiple Choice Questions (MCQs)

Which is a polynomial? (A) $x + 1/x$ (B) $\sqrt{x} + 2$ (C) $3x^2 – x + 5$ (D) $2/x^2$
Correct answer: (C) $3x^2 – x + 5$. Whole-number exponents.

Degree of $3x^2y^3 – xy$ is: (A) 2 (B) 3 (C) 5 (D) 6
Correct answer: (C) 5. 2 + 3.

A polynomial of degree 3 is: (A) Linear (B) Quadratic (C) Cubic (D) Biquadratic
Correct answer: (C) Cubic. Cubic.

$(x + 4)(x – 4)$ = (A) $x^2 – 16$ (B) $x^2 + 16$ (C) $x^2 – 8x + 16$ (D) $x^2 – 8$

Correct answer: (A) $x^2 - 16$. Difference of squares.

$(a - b)^2$ = (A) $a^2 - b^2$ (B) $a^2 - 2ab + b^2$ (C) $a^2 + 2ab + b^2$ (D) $a^2 + b^2$
Correct answer: (B) $a^2 - 2ab + b^2$. Identity.

101 × 99 = (A) 9999 (B) 10001 (C) 9899 (D) 10099
Correct answer: (A) 9999. $100^2 - 1$.

$x^2 + 6x + 9$ = (A) $(x + 3)^2$ (B) $(x - 3)^2$ (C) $(x + 9)(x + 1)$ (D) $(x + 6)(x + 3)$
Correct answer: (A) $(x + 3)^2$. Perfect square.

Factors of $x^2 + 7x + 10$ are: (A) $(x + 2)(x + 5)$ (B) $(x + 1)(x + 10)$ (C) $(x - 2)(x - 5)$ (D) $(x + 3)(x + 4)$
Correct answer: (A) $(x + 2)(x + 5)$. $2 \times 5 = 10$, $2 + 5 = 7$.

a^0 = (A) 0 (B) 1 (C) a (D) Undefined for $a \neq 0$
Correct answer: (B) 1. Zero index.

2^{-3} = (A) –8 (B) –6 (C) 1/8 (D) 8
Correct answer: (C) 1/8. $1/2^3$.

$(x^3)^4$ = (A) x^7 (B) x^{12} (C) x^{81} (D) x
Correct answer: (B) x^{12}. 3×4.

81^(1/2) = (A) 9 (B) 40.5 (C) 3 (D) 27
Correct answer: (A) 9. $\sqrt{81}$.

4,200 in scientific notation is: (A) 42×10^2 (B) 4.2×10^3 (C) 0.42×10^4 (D) 4.2×10^2
Correct answer: (B) 4.2×10^3. $1 \leq a < 10$.

0.0035 = (A) 3.5×10^{-3} (B) 3.5×10^3 (C) 35×10^{-4} (D) 3.5×10^{-2}
Correct answer: (A) 3.5×10^{-3}. Move point 3 places.

x + 3 = 7 is a(n): (A) Closed sentence (B) Open sentence (C) Identity (D) Polynomial
Correct answer: (B) Open sentence. Has a variable.

Quick Revision Summary

- Closed sentence: true/false; open sentence: contains a variable.
- Polynomial: whole-number exponents; degree → linear, quadratic, cubic, biquadratic.
- Product degree = sum of degrees; exact division leaves remainder 0.
- $(a \pm b)^2 = a^2 \pm 2ab + b^2$; $a^2 - b^2 = (a + b)(a - b)$; $(a \pm b)^3 = a^3 \pm 3ab(a \pm b) \pm b^3$.
- Factorize: common factor, grouping, identities, splitting the middle term.
- Exponent laws; $a^0 = 1$; $a^{-n} = 1/a^n$; a^(1/n) = $\sqrt[n]{a}$.

- Scientific notation: $a \times 10^n$, $1 \leq a < 10$.

Exam Tips

- Always look for a common factor first before trying other methods.
- Check a factorization by multiplying the factors back.
- Write identities at the top of your answer before using them.
- For negative indices, flip the base: $(a/b)^{-n} = (b/a)^n$.
- In scientific notation count how many places the decimal point moves.