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Mathematics Class 8 — Punjab Board (SNC)

UNIT 13

Probability

Key Concepts · Definitions · Short & Long Questions · MCQs · Diagrams · Exam Tips

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Unit 13: Probability

Probability measures how likely an event is to happen, on a scale from 0 (impossible) to 1 (certain). Unit 13 explains the language of chance, the difference between experimental and theoretical probability, and the complement rule P(not A) = 1 – P(A).

It then deals with combined events: mutually exclusive events and the addition (OR) rule, independent events and the multiplication (AND) rule, and listing outcomes with sample-space tables and tree diagrams.

Learning Objectives

- Describe events as impossible, unlikely, even chance, likely or certain, and place them on a probability scale.
- Find theoretical probability using P(E) = favourable outcomes ÷ total outcomes.
- Find experimental probability (relative frequency) from trials and expected frequency.
- Use P(not A) = 1 – P(A).
- Apply the addition rule to mutually exclusive events and the multiplication rule to independent events.
- List outcomes of combined events using tables and tree diagrams.

Key Concepts

13.1 Language and Scale of Probability

An experiment (such as tossing a coin) has possible results called outcomes; the set of all outcomes is the sample space. An event is one or more outcomes.

Probability lies between 0 and 1: 0 means impossible (getting 7 on an ordinary die), 1 means certain (the sun rising tomorrow), ½ means even chance (head on a fair coin). Values near 0 are unlikely and values near 1 are likely.

13.2 Theoretical Probability

When all outcomes are equally likely, P(E) = number of favourable outcomes ÷ total number of outcomes.

For one fair die: P(even) = 3/6 = ½; P(number greater than 4) = 2/6 = ⅓. For a bag of 3 red and 5 blue balls: P(red) = 3/8.

Complement rule: P(not A) = 1 – P(A). If P(rain) = 0.3, then P(no rain) = 0.7. The probabilities of all outcomes add up to 1.

13.3 Experimental Probability

Experimental probability (relative frequency) = number of times the event happened ÷ number of trials. If a coin shows heads 28 times in 50 tosses, the experimental probability of a head is 28/50 =

0.56.

As the number of trials increases, experimental probability usually gets closer to the theoretical value.

Expected frequency = probability × number of trials. A fair die rolled 120 times is expected to show six about 120 × ⅙ = 20 times.

13.4 Mutually Exclusive Events (OR rule)

Events are mutually exclusive if they cannot happen together, so P(A and B) = 0. Getting 1 and getting 6 on one roll of a die are mutually exclusive.

Addition rule: P(A or B) = P(A) + P(B). So P(1 or 6) = ⅙ + ⅙ = ⅓.

13.5 Independent Events (AND rule)

Events are independent if one does not affect the other, such as tossing a coin and rolling a die.

Multiplication rule: P(A and B) = P(A) × P(B). P(head and six) = ½ × ⅙ = 1/12. Drawing a red ball twice with replacement from 3 red and 5 blue: (3/8) × (3/8) = 9/64.

13.6 Combined Events: Tables and Tree Diagrams

A tree diagram shows each stage as branches labelled with probabilities. Multiply along a path to get the probability of that outcome, and add the paths for 'or'. Two coin tosses give HH, HT, TH, TT, each ¼; P(at least one head) = ¾.

A sample-space table lists all outcomes of two dice: 36 equally likely pairs. P(sum = 7) = 6/36 = ⅙; P(a double) = 6/36 = ⅙; P(sum = 12) = 1/36.

Important Definitions

Experiment
An action with uncertain results, such as rolling a die.

Outcome
One possible result of an experiment.

Sample space
The set of all possible outcomes.

Event
One or more outcomes of an experiment.

Equally likely outcomes
Outcomes that have the same chance of happening.

Theoretical probability

Favourable outcomes ÷ total outcomes.

Experimental probability
Number of successes ÷ number of trials.

Complementary event
The event 'not A'; P(not A) = 1 – P(A).

Mutually exclusive events
Events that cannot happen at the same time.

Independent events
Events where one does not affect the other.

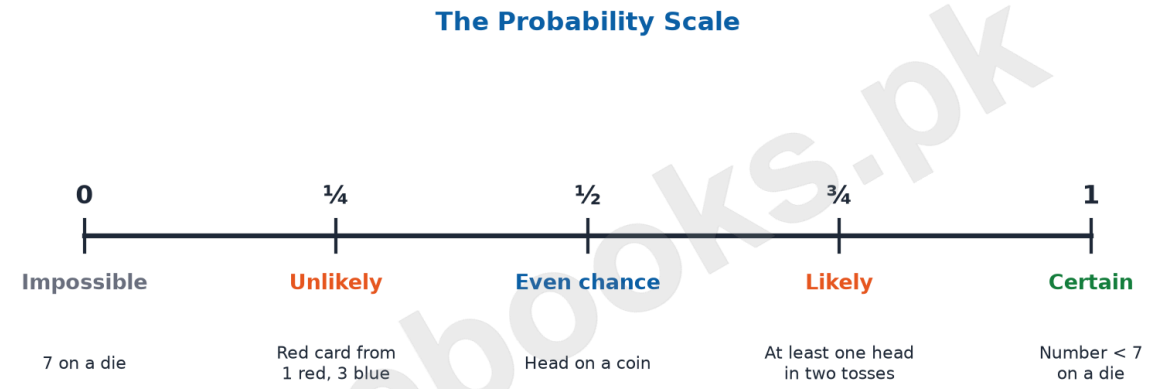
Tree diagram
A branching diagram showing outcomes of combined events.

Key Facts at a Glance

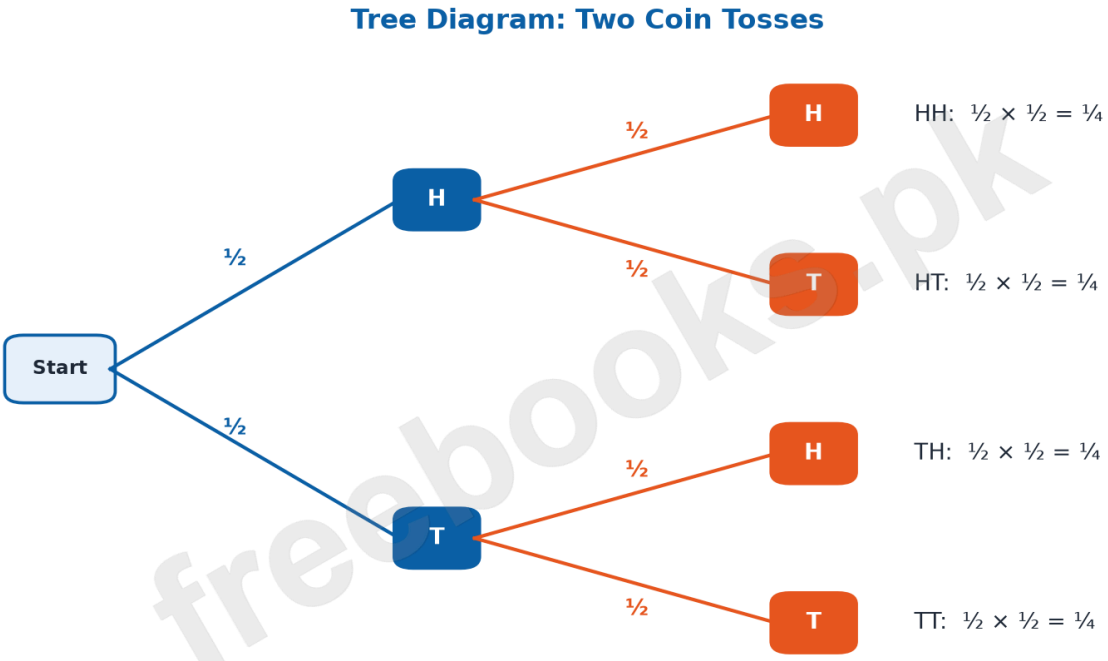
Rule / case	Result
Probability range	$0 \le P(E) \le 1$
Theoretical probability	Favourable ÷ total
Complement	P(not A) = 1 – P(A)
Mutually exclusive (OR)	P(A or B) = P(A) + P(B)
Independent (AND)	P(A and B) = P(A) × P(B)
Expected frequency	P × number of trials
P(even) on a die	½
P(head and six)	1/12
Two coins: P(at least one head)	¾
Two dice: P(sum 7)	⅙
Two dice: P(sum 12)	1/36
28 heads in 50 tosses	0.56

Diagrams

Probability scale: A scale from 0 to 1 marked impossible, unlikely, even chance, likely and certain with examples.



Tree diagram for two coins: A tree diagram of two coin tosses with probabilities on each branch.



Sample space of two dice: A 6 × 6 table of sums of two dice with the six ways of scoring 7 highlighted.

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Two Dice: 36 Outcomes — P(sum = 7) = 6/36 = ⅙

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

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Short Questions and Answers

What is the probability of an impossible event?
0.

Give an example of a certain event.
Getting a number less than 7 on an ordinary die.

Find P(5) for a fair die.
⅙.

Find P(prime) for a fair die.

2, 3, 5 → 3/6 = ½.

A bag has 4 green and 6 yellow balls. Find P(green).
4/10 = 2/5.

If P(win) = 0.35, find P(not win).
0.65.

A spinner landed on red 18 times in 60 spins. Find the experimental probability.
18/60 = 0.3.

A fair coin is tossed 200 times. How many heads are expected?
100.

What are mutually exclusive events?
Events that cannot happen at the same time.

Find P(2 or 3) on one roll of a die.
⅙ + ⅙ = ⅓.

What are independent events?
Events where the result of one does not affect the other.

Find P(tail and even number) for a coin and a die.
½ × ½ = ¼.

Two coins are tossed. Find P(two tails).
¼.

Two dice are rolled. How many outcomes are there?
36.

Two dice are rolled. Find P(a double).
6/36 = ⅙.

Long Questions and Answers

Explain the probability scale.

What range does probability have?

From 0 to 1, written as fractions, decimals or percentages.

What do 0 and 1 mean?

0 means impossible and 1 means certain.

What does ½ mean?

Even chance, like a head on a fair coin.

Place 'a day of the week starting with S' on the scale.

Saturday and Sunday: 2/7 ≈ 0.29, which is unlikely.

Find theoretical probabilities for a fair die.

List the sample space.

{1, 2, 3, 4, 5, 6}, six equally likely outcomes.

P(odd)

1, 3, 5 → 3/6 = ½.

P(multiple of 3)

3, 6 → 2/6 = ⅓.

P(not a multiple of 3)

1 – ⅓ = ⅔.

Compare experimental and theoretical probability.

Definitions

Experimental = successes ÷ trials; theoretical = favourable ÷ total outcomes.

Example

A die rolled 60 times showed four 14 times: experimental P(4) = 14/60 ≈ 0.23; theoretical ⅙ ≈ 0.17.

Why do they differ?

Results of a small number of trials vary by chance.

What happens with more trials?

The experimental value usually comes closer to the theoretical value.

Explain mutually exclusive events and the addition rule.

Meaning

Two events that cannot occur together; P(A and B) = 0.

Addition rule

P(A or B) = P(A) + P(B).

A card is picked from cards numbered 1–10. Find P(less than 3 or greater than 8).

2/10 + 2/10 = 2/5.

Are 'even' and 'greater than 4' on a die mutually exclusive?

No; 6 is in both, so the simple addition rule does not apply.

Explain independent events and the multiplication rule.

Meaning

The outcome of one event does not change the probability of the other.

Multiplication rule

P(A and B) = P(A) × P(B).

A coin is tossed and a die rolled. Find P(head and number less than 3).

½ × ⅓ = ⅙.

A ball is drawn from 2 red and 3 white, replaced, and drawn again. Find P(both white).

(3/5) × (3/5) = 9/25.

Use a tree diagram for two tosses of a coin.

Draw the first stage.

Two branches, H and T, each with probability ½.

Draw the second stage.

From each branch, two more branches H and T, each ½, giving HH, HT, TH, TT.

Find P(exactly one head).

HT + TH = ¼ + ¼ = ½.

Find P(at least one head).

1 – P(TT) = 1 – ¼ = ¾.

Multiple Choice Questions (MCQs)

Probability of a certain event is: (A) 0 (B) ½ (C) 1 (D) 2
Correct answer: (C) 1. Certain = 1.

Which cannot be a probability? (A) 0.7 (B) ⅗ (C) 1.2 (D) 0
Correct answer: (C) 1.2. Greater than 1.

P(head) on a fair coin: (A) 0 (B) ½ (C) 1 (D) ¼
Correct answer: (B) ½. Even chance.

P(6) on a fair die: (A) ⅙ (B) ⅓ (C) ½ (D) 1
Correct answer: (A) ⅙. One of six.

If P(A) = 0.4, P(not A) = (A) 0.4 (B) 0.6 (C) 1.4 (D) 0
Correct answer: (B) 0.6. 1 – 0.4.

A bag has 2 red, 3 blue, 5 green. P(blue) = (A) 0.2 (B) 0.3 (C) 0.5 (D) 0.6
Correct answer: (B) 0.3. 3/10.

30 heads in 80 tosses gives experimental P(head): (A) 0.375 (B) 0.5 (C) 0.3 (D) 0.8
Correct answer: (A) 0.375. 30/80.

Expected number of sixes in 300 rolls: (A) 30 (B) 50 (C) 60 (D) 100
Correct answer: (B) 50. 300 × ⅙.

Mutually exclusive events have P(A and B) = (A) 1 (B) P(A)P(B) (C) 0 (D) ½
Correct answer: (C) 0. Cannot occur together.

For mutually exclusive events P(A or B) = (A) P(A)P(B) (B) P(A) + P(B) (C) P(A) – P(B) (D) 1
Correct answer: (B) P(A) + P(B). Addition rule.

For independent events P(A and B) = (A) P(A) + P(B) (B) P(A) × P(B) (C) 0 (D) 1
Correct answer: (B) P(A) × P(B). Multiplication rule.

Two coins tossed: number of outcomes: (A) 2 (B) 3 (C) 4 (D) 6
Correct answer: (C) 4. HH, HT, TH, TT.

Two dice: P(sum = 7): (A) 1/36 (B) ⅙ (C) 7/36 (D) ½
Correct answer: (B) ⅙. 6/36.

Two dice: P(sum = 12): (A) 1/36 (B) 1/12 (C) ⅙ (D) 1/18
Correct answer: (A) 1/36. Only (6, 6).

On a tree diagram, probabilities along a path are: (A) Added (B) Multiplied (C) Subtracted (D) Divided
Correct answer: (B) Multiplied. AND rule.

Quick Revision Summary

- $0 \le P \le 1$: impossible 0, even chance ½, certain 1.
- Theoretical P = favourable ÷ total; P(not A) = 1 – P(A).
- Experimental P = successes ÷ trials; expected frequency = P × trials.
- Mutually exclusive: P(A or B) = P(A) + P(B).
- Independent: P(A and B) = P(A) × P(B).
- Tree diagrams: multiply along branches, add the paths.

Exam Tips

- Write the sample space first; count carefully.
- Leave answers as simplified fractions unless decimals are asked for.

- Use 1 – P for 'at least one' questions.
- Check that branch probabilities from each point add up to 1.
- Decide first whether events are 'or' (add) or 'and' (multiply).