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Mathematics Class 8 — Punjab Board (SNC)

UNIT 10

Mensuration

Key Concepts · Definitions · Short & Long Questions · MCQs · Diagrams · Exam Tips

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Unit 10: Mensuration

Mensuration is the measurement of lengths, areas and volumes. Unit 10 starts with Pythagoras theorem for right-angled triangles, then moves to the parts of a circle, the length of an arc and the area of a sector.

The second half deals with solid shapes: the surface area and volume of a pyramid, sphere, hemisphere and cone, and of composite solids made by joining these shapes. Unless told otherwise, these notes take $\pi = 22/7$.

Learning Objectives

- State Pythagoras theorem and use it to find a missing side of a right-angled triangle.
- Identify the parts of a circle: radius, diameter, chord, arc, sector, segment and tangent.
- Calculate the length of an arc and the area of a sector.
- Find the surface area and volume of a pyramid.
- Find the surface area and volume of a sphere, hemisphere and cone.
- Solve problems on composite solids.

Key Concepts

10.1 Pythagoras Theorem

In a right-angled triangle, the square of the hypotenuse equals the sum of the squares of the other two sides: $c^2 = a^2 + b^2$, where c is the side opposite the right angle.

Example: legs 6 cm and 8 cm give $c = \sqrt{(36 + 64)} = \sqrt{100} = 10$ cm. To find a leg, subtract: $a = \sqrt{(c^2 – b^2)}$.

Sets of whole numbers that satisfy the theorem are called Pythagorean triples: (3, 4, 5), (5, 12, 13), (8, 15, 17). The converse is also true: if $c^2 = a^2 + b^2$, the triangle is right-angled.

10.2 Circle, Arc and Sector

Radius: centre to the circle. Diameter: a chord through the centre ($d = 2r$). Chord: a segment joining two points on the circle. Arc: a part of the circumference. Sector: the region between two radii and an arc. Segment: the region between a chord and an arc. Tangent: a line touching the circle at one point.

Circumference $= 2\pi r$ and area $= \pi r^2$.

For a central angle x°: arc length $L = (x/360) \times 2\pi r$ and sector area $A = (x/360) \times \pi r^2$. For $r = 7$ cm and $x = 90°$: $L = ¼ \times 44 = 11$ cm and $A = ¼ \times 154 = 38.5$ cm².

10.3 Pyramid

A pyramid has a polygon base and triangular faces meeting at a point called the apex. The height h is the perpendicular from the apex to the base; the slant height l is the height of a triangular face.

Surface area = base area + area of all triangular faces, each face = ½ × base edge × slant height. Volume = ⅓ × base area × h.

Example: square base 12 cm, height 8 cm. Slant height l = $\sqrt{(8^2 + 6^2)}$ = 10 cm. Faces = 4 × ½ × 12 × 10 = 240 cm²; total surface = 240 + 144 = 384 cm². Volume = ⅓ × 144 × 8 = 384 cm³.

10.4 Sphere and Hemisphere

Sphere: surface area = $4\pi r^2$; volume = $(4/3)\pi r^3$. For r = 7 cm: surface = 616 cm² and volume ≈ 1437.33 cm³.

A hemisphere is half a sphere. Curved surface = $2\pi r^2$; total surface (with the flat circle) = $3\pi r^2$; volume = $(2/3)\pi r^3$. For r = 7 cm: curved 308 cm², total 462 cm², volume ≈ 718.67 cm³.

10.5 Cone

A cone has a circular base and an apex. Radius r, height h and slant height l are related by $l^2 = r^2 + h^2$.

Curved surface = πrl; total surface = $\pi r(r + l)$; volume = $\frac{1}{3}\pi r^2 h$.

Example: r = 7 cm, h = 24 cm, so l = 25 cm. Curved surface = 22/7 × 7 × 25 = 550 cm²; total = 22/7 × 7 × 32 = 704 cm²; volume = ⅓ × 22/7 × 49 × 24 = 1232 cm³.

10.6 Composite Solids

A composite solid is made by joining simple solids. Add the volumes of the parts. For surface area, add only the outside surfaces and leave out joined faces.

Example: a cone of radius 7 cm and height 6 cm on a hemisphere of radius 7 cm. Volume = 308 + 718.67 ≈ 1026.67 cm³.

Important Definitions

Hypotenuse
The longest side of a right-angled triangle, opposite the right angle.

Pythagorean triple
Three whole numbers a, b, c with $a^2 + b^2 = c^2$.

Chord
A line segment joining two points on a circle.

Arc
A part of the circumference of a circle.

Sector
The region bounded by two radii and an arc.

Segment
The region bounded by a chord and an arc.

Tangent
A line that touches a circle at exactly one point.

Apex
The top point of a pyramid or cone.

Slant height
The distance from the apex down the slanting surface to the base edge.

Hemisphere
Half of a sphere cut through its centre.

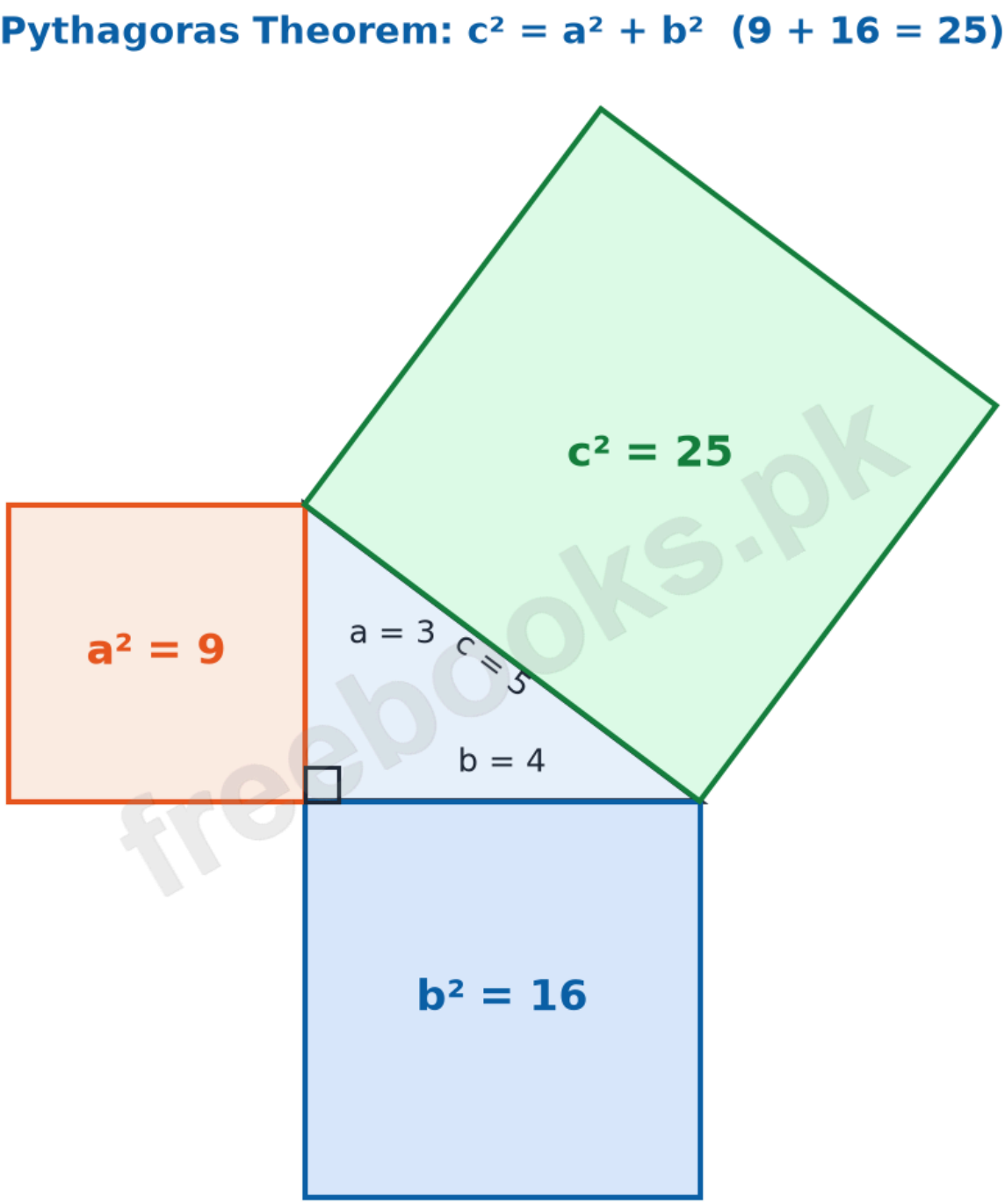
Composite solid
A solid made by combining two or more simple solids.

Key Facts at a Glance

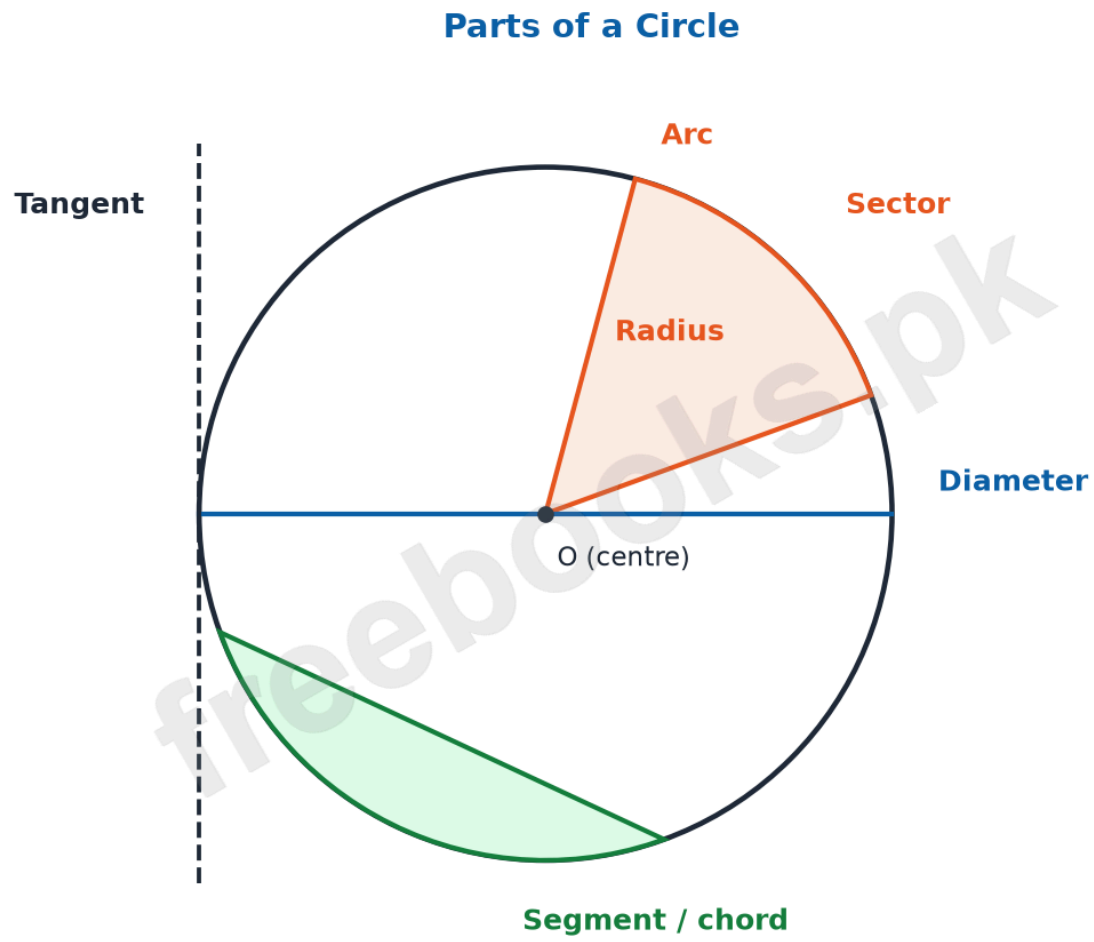
Quantity	Formula
Pythagoras	$c^2 = a^2 + b^2$
Arc length	$(x/360) \times 2\pi r$
Sector area	$(x/360) \times \pi r^2$
Pyramid surface area	Base + Σ ½ × edge × slant height
Pyramid volume	⅓ × base area × h
Sphere surface area	$4\pi r^2$
Sphere volume	$(4/3)\pi r^3$
Hemisphere total surface	$3\pi r^2$
Hemisphere volume	$(2/3)\pi r^3$
Cone slant height	$l^2 = r^2 + h^2$
Cone surfaces	Curved πrl; total $\pi r(r + l)$
Cone volume	$\frac{1}{3}\pi r^2 h$

Diagrams

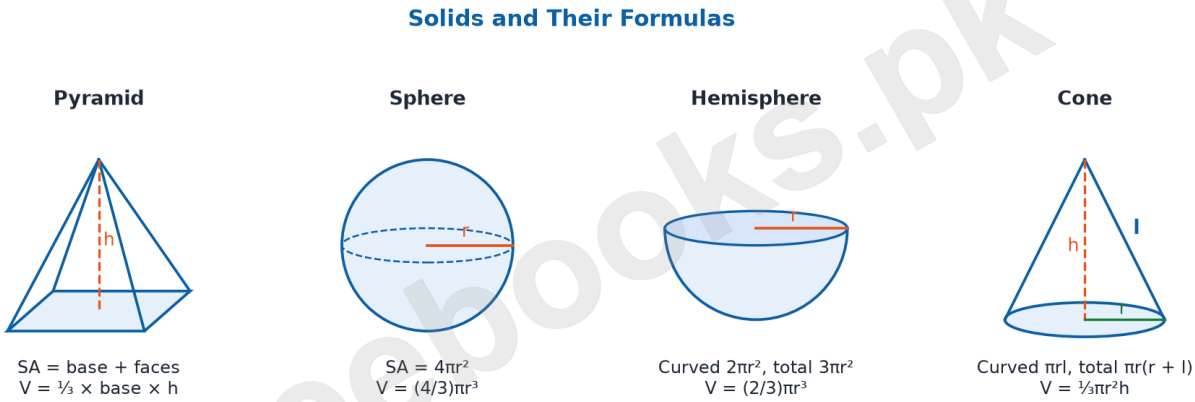
Pythagoras theorem: A right-angled triangle with squares on each side showing $c^2 = a^2 + b^2$.



Parts of a circle: A circle labelled with radius, diameter, chord, arc, sector, segment and tangent.



Solids and formulas: A pyramid, sphere, hemisphere and cone with their surface area and volume formulas.



Short Questions and Answers

State Pythagoras theorem.
In a right-angled triangle, hypotenuse² = sum of squares of the other two sides.

Find the hypotenuse if the legs are 5 cm and 12 cm.
√169 = 13 cm.

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A ladder 17 m long reaches 15 m up a wall. How far is its foot from the wall?
$\sqrt{(289 - 225)} = 8$ m.

Are 7, 24, 25 the sides of a right triangle?
Yes: $49 + 576 = 625$.

What is the difference between a chord and a diameter?
A diameter is a chord that passes through the centre.

Find the arc length for r = 14 cm, x = 45°.
$\frac{1}{8} \times 2 \times 22/7 \times 14 = 11$ cm.

Find the sector area for r = 14 cm, x = 60°.
$\frac{1}{6} \times 22/7 \times 196 \approx 102.67$ cm².

Find the volume of a pyramid with base area 50 cm² and height 9 cm.
$\frac{1}{3} \times 50 \times 9 = 150$ cm³.

Find the slant height of a square pyramid of base 10 cm and height 12 cm.
$\sqrt{(144 + 25)} = 13$ cm.

Find the surface area of a sphere of radius 14 cm.
$4 \times 22/7 \times 196 = 2464$ cm².

Find the volume of a sphere of radius 3 cm (π = 3.14).
$4/3 \times 3.14 \times 27 = 113.04$ cm³.

Find the total surface of a hemisphere of radius 14 cm.
$3 \times 22/7 \times 196 = 1848$ cm².

Find the slant height of a cone with r = 6 cm, h = 8 cm.
10 cm.

Find the curved surface of a cone with r = 7 cm, l = 10 cm.
$22/7 \times 7 \times 10 = 220$ cm².

Find the volume of a cone with r = 3 cm, h = 7 cm.
$\frac{1}{3} \times 22/7 \times 9 \times 7 = 66$ cm³.

Long Questions and Answers

State and apply Pythagoras theorem.

State the theorem.

In a right-angled triangle, $c^2 = a^2 + b^2$, where c is the hypotenuse.

Find the missing leg if c = 26 cm and a = 10 cm.

$b = \sqrt{(676 - 100)} = \sqrt{576} = 24$ cm.

Find the diagonal of a rectangle 9 cm by 12 cm.

$\sqrt{(81 + 144)} = \sqrt{225} = 15$ cm.

What is the converse?

If the square of one side equals the sum of the squares of the other two, the triangle is right-angled.

Explain arc length and sector area.

What fraction of the circle is a sector?

x/360 of the whole circle, where x is the central angle.

Write the formulas.

$L = (x/360) \times 2\pi r$ and $A = (x/360) \times \pi r^2$.

Find L and A for r = 21 cm, x = 120°.

$L = ⅓ \times 132 = 44$ cm; $A = ⅓ \times 1386 = 462$ cm².

Find the perimeter of that sector.

44 + 21 + 21 = 86 cm.

Find the surface area and volume of a pyramid.

Describe a square pyramid.

A square base with four triangular faces meeting at the apex.

Base 16 cm, height 6 cm: find the slant height.

$\sqrt{(6^2 + 8^2)} = 10$ cm.

Find its total surface area.

Faces $4 \times ½ \times 16 \times 10 = 320$ cm²; base 256 cm²; total 576 cm².

Find its volume.

$⅓ \times 256 \times 6 = 512$ cm³.

Explain the sphere and the hemisphere.

Formulas for a sphere

Surface area $4\pi r^2$ and volume $(4/3)\pi r^3$.

Formulas for a hemisphere

Curved $2\pi r^2$, total $3\pi r^2$, volume $(2/3)\pi r^3$.

A ball has radius 21 cm. Find its surface area.

$4 \times 22/7 \times 441 = 5544$ cm^2.

Find the volume of a hemispherical bowl of radius 21 cm.

$2/3 \times 22/7 \times 9261 = 19404$ cm^3.

Find the surface area and volume of a cone.

How are r, h and l related?

$l^2 = r^2 + h^2$, by Pythagoras theorem.

For r = 5 cm and h = 12 cm, find l.

l = 13 cm.

Find its curved and total surface (π = 3.14).

Curved $3.14 \times 5 \times 13 = 204.1$ cm^2; total $3.14 \times 5 \times 18 = 282.6$ cm^2.

Find its volume.

$⅓ \times 3.14 \times 25 \times 12 = 314$ cm^3.

Solve a problem on a composite solid.

Describe the solid.

A toy is a cone of radius 7 cm and height 24 cm standing on a hemisphere of the same radius.

Volume of the cone

$⅓ \times 22/7 \times 49 \times 24 = 1232$ cm^3.

Volume of the hemisphere

$2/3 \times 22/7 \times 343 \approx 718.67$ cm^3.

Total volume and outer surface

Volume ≈ 1950.67 cm^3; surface = cone curved 550 + hemisphere curved 308 = 858 cm^2.

Multiple Choice Questions (MCQs)

In a right triangle, the longest side is the: (A) Base (B) Perpendicular (C) Hypotenuse (D) Median

Correct answer: (C) Hypotenuse. Opposite the right angle.

Legs 9 and 12 give hypotenuse: (A) 13 (B) 15 (C) 21 (D) 17

Correct answer: (B) 15. √225.

Which is a Pythagorean triple? (A) (2, 3, 4) (B) (4, 5, 6) (C) (8, 15, 17) (D) (6, 7, 8)

Correct answer: (C) (8, 15, 17). 64 + 225 = 289.

A chord through the centre is a: (A) Radius (B) Diameter (C) Tangent (D) Arc
Correct answer: (B) Diameter. Longest chord.

Region between two radii and an arc: (A) Segment (B) Sector (C) Chord (D) Tangent
Correct answer: (B) Sector. Sector.

Arc length formula is: (A) (x/360)πr² (B) (x/360)2πr (C) 2πr² (D) πr/x
Correct answer: (B) (x/360)2πr. Fraction of circumference.

Volume of a pyramid: (A) Base × h (B) ½ × base × h (C) ⅓ × base × h (D) ¼ × base × h
Correct answer: (C) ⅓ × base × h. One third.

Surface area of a sphere: (A) 2πr² (B) 3πr² (C) 4πr² (D) πr²
Correct answer: (C) 4πr². 4πr².

Volume of a sphere: (A) (4/3)πr³ (B) (2/3)πr³ (C) ⅓πr³ (D) 4πr³
Correct answer: (A) (4/3)πr³. (4/3)πr³.

Total surface of a hemisphere: (A) 2πr² (B) 3πr² (C) 4πr² (D) πr²
Correct answer: (B) 3πr². 2πr² + πr².

Curved surface of a cone: (A) πrh (B) πrl (C) 2πrl (D) πr²
Correct answer: (B) πrl. πrl.

Volume of a cone: (A) πr²h (B) ⅓πr²h (C) ½πr²h (D) (4/3)πr²h
Correct answer: (B) ⅓πr²h. One third of cylinder.

Cone with r = 3, h = 4 has slant height: (A) 5 (B) 7 (C) 6 (D) 12
Correct answer: (A) 5. √25.

Sector area for x = 180° is: (A) πr² (B) ½πr² (C) ¼πr² (D) 2πr²
Correct answer: (B) ½πr². Half the circle.

A line touching a circle at one point is a: (A) Chord (B) Secant (C) Tangent (D) Radius
Correct answer: (C) Tangent. Tangent.

Quick Revision Summary

- $c^2 = a^2 + b^2$; triples (3,4,5), (5,12,13), (8,15,17).
- Arc L = (x/360)2πr; sector A = (x/360)πr².
- Pyramid: SA = base + triangular faces; V = ⅓ × base × h; slant from Pythagoras.
- Sphere 4πr², (4/3)πr³; hemisphere 2πr² / 3πr², (2/3)πr³.
- Cone $l^2 = r^2 + h^2$; curved πrl; total πr(r + l); V = ⅓πr²h.
- Composite: add volumes; count only outer surfaces.

Exam Tips

- Draw and label a sketch before using any formula.
- Check whether the question uses $\pi = 22/7$ or 3.14.
- Use the slant height for surface area and the vertical height for volume.
- Write units: cm for length, cm^2 for area, cm^3 for volume.
- Choose r as a multiple of 7 when estimating with 22/7 to keep working simple.