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MATHEMATICS CLASS 8 — PUNJAB BOARD (SNC)

Mathematics Class 8 Complete Notes

All 13 Units — English Medium

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FBPK-MATH8-2026-11-FULLBOOK

Unit 1: Real Numbers

Unit 1 introduces the real number system, the set of all numbers we use for counting, measuring and calculating. It starts from natural numbers and builds up through whole numbers, integers and rational numbers, then adds irrational numbers such as $\sqrt{2}$ and π to complete the set of real numbers ($\mathbb{R}$).

These notes cover the types of numbers, terminating and recurring decimals, showing numbers on a number line, absolute value and distance between numbers, word problems with fractions and decimals, the properties of real numbers under addition and multiplication, and the order (inequality) properties.

Learning Objectives

- Tell the difference between rational and irrational numbers.
- Classify decimals as terminating, recurring or non-terminating non-recurring.
- Represent fractions and negative numbers on a number line.
- Find the absolute value of a number and the distance between two numbers.
- Solve word problems involving fractions and decimals.
- State and use the properties and order properties of real numbers.

Key Concepts

1.1 Types of Numbers

Natural numbers start from 1: $N = \{1, 2, 3, ...\}$. Adding 0 gives whole numbers: $W = \{0, 1, 2, 3, ...\}$. Adding the negatives gives integers: $Z = \{..., -3, -2, -1, 0, 1, 2, 3, ...\}$.

A rational number can be written as p/q where p and q are integers and $q \neq 0$, for example $3/4$, $-5/2$, 0 and 12 (= 12/1). The set of rational numbers is Q. Every integer is a rational number.

In decimal form a rational number is either terminating (finite digits after the point, e.g., 0.375 = 3/8) or non-terminating recurring (a digit or block repeats forever, e.g., 4/11 = 0.3636...).

An irrational number (set Q') cannot be written as p/q; its decimal never ends and never repeats, e.g., $\sqrt{2} = 1.41421...$, $\sqrt{3}$, $\pi = 3.14159...$. The square root of a number that is not a perfect square is irrational. Note that 22/7 is rational and is only an approximation of π ($\pi \approx 22/7$).

Rational and irrational numbers together make the real numbers: $\mathbb{R} = Q \cup Q'$.

1.2 Real Numbers on a Number Line

A number line extends in both directions from 0: positive numbers to the right and negative numbers to the left.

To show l/m, divide each unit into m equal parts and count l parts from 0 (to the right if positive, to the left if negative). For example, $7/5 = 1 + 2/5$, so divide the gap from 1 to 2 into five parts and mark the second point. For $-1/3$, divide the gap from 0 to -1 into three parts and mark the first

point to the left of 0.

1.3 Absolute Value

The absolute value (modulus) of a real number x, written |x|, is its distance from 0 on the number line, so it is never negative: |6| = 6 and |–6| = 6.

The distance between two numbers a and b is |a – b|. For example, the distance between –3 and 2 is |–3 – 2| = |–5| = 5 units; the order of subtraction does not matter because |2 – (–3)| is also 5.

1.4 Word Problems with Fractions and Decimals

To find how many equal pieces fit in a length, divide: a ribbon of 15/2 m cut into pieces of 1/4 m gives 15/2 ÷ 1/4 = 15/2 × 4 = 30 pieces.

Decimals are added, subtracted, multiplied and divided in the usual way, keeping the decimal points lined up when adding or subtracting. Example: 325.650 km + 410.350 km = 736 km; over 30 days the daily average is 325.650 ÷ 30 = 10.855 km.

1.5 Properties of Real Numbers

Closure: the sum and product of two real numbers are real numbers. Associative: (a + b) + c = a + (b + c) and (ab)c = a(bc). Commutative: a + b = b + a and ab = ba.

Identity: 0 is the additive identity (a + 0 = a) and 1 is the multiplicative identity (a × 1 = a). Inverses: the additive inverse of a is –a (a + (–a) = 0); for a ≠ 0 the multiplicative inverse is 1/a (a × 1/a = 1). Zero has no multiplicative inverse.

Distributive: a(b + c) = ab + ac, for example 2(3 + 4) = 2×3 + 2×4 = 14.

1.6 Order Properties (Inequalities)

Trichotomy: for any two real numbers exactly one of a < b, a = b or a > b is true. Transitive: if a > b and b > c then a > c.

Adding or subtracting the same number on both sides keeps the inequality sign: if a < b then a + c < b + c and a – c < b – c.

Multiplying or dividing by a positive number keeps the sign, but by a negative number reverses it: 5 < 8 gives 10 < 16 (× 2) but –10 > –16 (× –2).

For two numbers of the same sign, taking reciprocals reverses the order: 2 < 4 gives 1/2 > 1/4.

Important Definitions

Natural numbers
Counting numbers 1, 2, 3, ...; set N.

Whole numbers
Natural numbers together with 0; set W.

Integers
Whole numbers and their negatives; set Z.

Rational number
A number that can be written as p/q, where p, q are integers and $q \neq 0$.

Irrational number
A number that cannot be written as p/q; its decimal is non-terminating and non-recurring.

Real numbers
All rational and irrational numbers together; set $\mathbb{R}$.

Terminating decimal
A decimal with a finite number of digits after the point.

Recurring decimal
A non-terminating decimal in which a digit or group of digits repeats.

Absolute value
The distance of a number from zero, written $|x|$; never negative.

Additive identity
0, because $a + 0 = a$.

Multiplicative identity
1, because $a \times 1 = a$.

Additive inverse
The number $-a$, which added to a gives 0.

Multiplicative inverse
The number $1/a$ ($a \neq 0$), which multiplied by a gives 1.

Closure property
Combining two real numbers by addition or multiplication gives a real number.

Trichotomy property
Exactly one of $a < b$, $a = b$, $a > b$ holds.

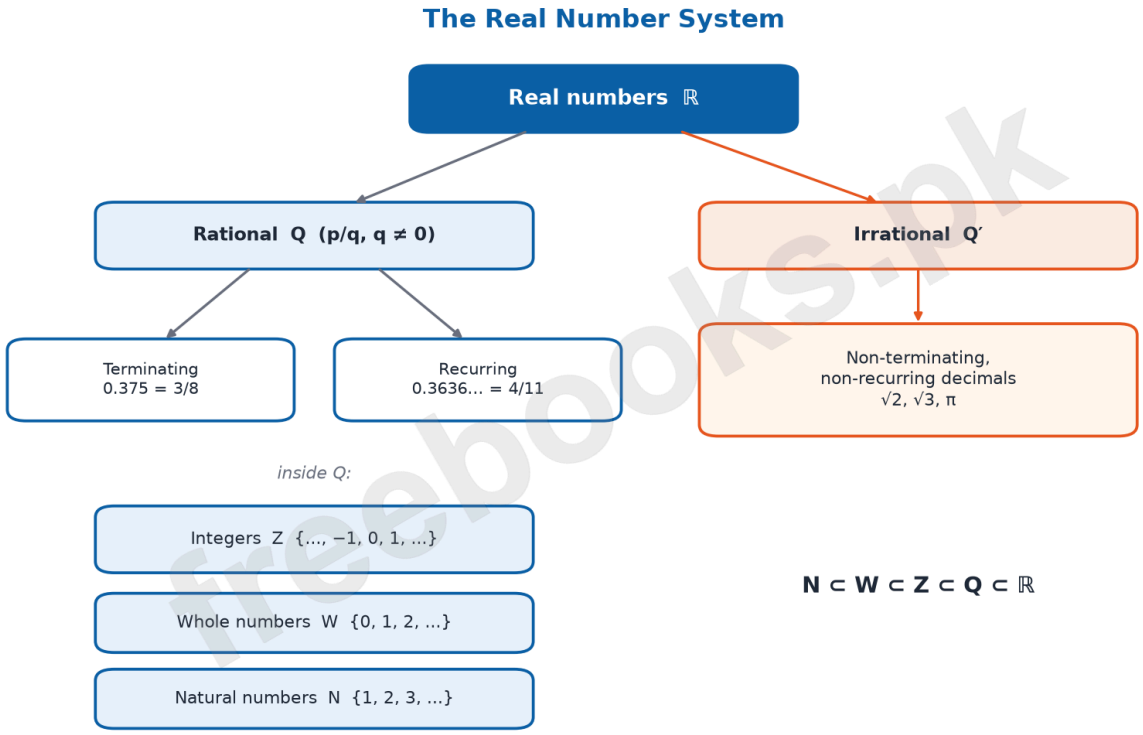
Key Facts at a Glance

Property / item	Example
$N \subset W \subset Z \subset Q \subset \mathbb{R}$	Each set lies inside the next

Terminating decimal	3/8 = 0.375
Recurring decimal	4/11 = 0.3636...
Irrational numbers	√2, √3, π
π and 22/7	π ≈ 22/7 (not equal)
Absolute value	|–7| = 7
Distance between –3 and 2	|–3 – 2| = 5
Commutative (+)	13 + 7 = 7 + 13
Associative (×)	(2 × 3) × 4 = 2 × (3 × 4)
Distributive	2(3 + 4) = 2×3 + 2×4
Inverse of 3/5	Additive –3/5; multiplicative 5/3
Multiply by negative	Inequality sign reverses

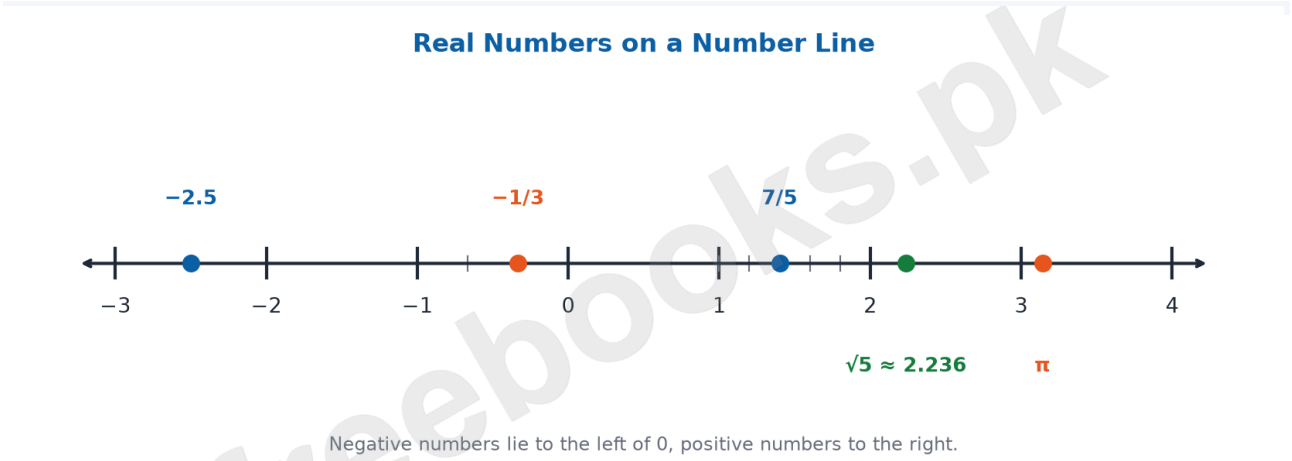
Diagrams

The real number system: A tree showing how natural, whole, integer, rational and irrational numbers make up the real numbers.



Real numbers on a number line: A number line marking –2.5, –1/3, 7/5, √5 and π.

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Properties of real numbers: A comparison chart of the properties under addition and multiplication.

Properties of Real Numbers

Property	Addition	Multiplication
Closure	$a + b \in \mathbb{R}$	$a \times b \in \mathbb{R}$
Associative	$(a + b) + c = a + (b + c)$	$(ab)c = a(bc)$
Commutative	$a + b = b + a$	$ab = ba$
Identity	$a + 0 = a \quad (0)$	$a \times 1 = a \quad (1)$
Inverse	$a + (-a) = 0$	$a \times 1/a = 1 \quad (a \neq 0)$

Distributive: $a(b + c) = ab + ac$

Short Questions and Answers

Is 0 a rational number? Why?

Yes. It can be written as 0/1, which has the form p/q with $q \neq 0$.

Why is √9 rational but √10 irrational?

$\sqrt{9} = 3 = 3/1$ is rational; 10 is not a perfect square, so $\sqrt{10}$ has a non-ending, non-repeating decimal.

Is 22/7 equal to π?

No. 22/7 is rational and only close to π; we write $\pi \approx 22/7$.

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Classify 7/20 and 5/6 as terminating or recurring decimals.
7/20 = 0.35 is terminating; 5/6 = 0.8333... is recurring.

How would you show –2/5 on a number line?
Divide the gap from 0 to –1 into 5 equal parts and mark the second point to the left of 0.

Find |–18| and |18|.
Both equal 18, because absolute value is distance from zero.

Find the distance between –9 and –4.
|–9 – (–4)| = |–5| = 5 units.

A 6 m pipe is cut into pieces of 3/4 m. How many pieces are obtained?
6 ÷ 3/4 = 6 × 4/3 = 8 pieces.

Write the additive and multiplicative inverses of –4.
Additive inverse 4; multiplicative inverse –1/4.

Why has 0 no multiplicative inverse?
There is no number that gives 1 when multiplied by 0, since 0 × anything = 0.

Name the property: $\sqrt{2} + \sqrt{7} = \sqrt{7} + \sqrt{2}$.
Commutative property of addition.

Name the property: $5(x + 2) = 5x + 10$.
Distributive property of multiplication over addition.

If $a > b$ and $b > 9$, what can you say about a?
$a > 9$, by the transitive property.

What happens to $3 < 7$ when both sides are multiplied by –1?
The sign reverses: $–3 > –7$.

If $3 < 5$, compare 1/3 and 1/5.
1/3 > 1/5, because reciprocals of positive numbers reverse the order.

Long Questions and Answers

Describe the different types of numbers that make up the real numbers.

What are natural, whole numbers and integers?

Natural numbers are 1, 2, 3, ...; whole numbers add 0; integers add the negative numbers –1, –2, –3, ... Each set contains the one before it.

What is a rational number?

Any number of the form p/q with integers p, q and q ≠ 0, such as 2/3, –7 and 0.25. Its decimal either terminates or recurs.

What is an irrational number?

A number such as √2 or π that cannot be written as p/q. Its decimal neither ends nor repeats.

How do they form the real numbers?

All rational and irrational numbers together form ℝ, and every real number has a point on the number line.

Explain terminating and recurring decimals with examples.

What is a terminating decimal?

A decimal with a finite number of digits after the point, e.g., 3/8 = 0.375 and 2/5 = 0.4. It can always be written as a fraction.

What is a recurring decimal?

A non-ending decimal with a repeating digit or block, e.g., 2/9 = 0.222... and 4/11 = 0.3636....

Are recurring decimals rational?

Yes. Every recurring decimal can be written as a fraction, e.g., 0.333... = 1/3.

How is an irrational decimal different?

It never ends and has no repeating block, like √2 = 1.41421356....

Explain absolute value and how to find the distance between two numbers.

What does |x| mean?

The distance of x from 0 on the number line, so it is always zero or positive.

Give examples.

|12| = 12, |–12| = 12 and |0| = 0.

How is distance between two numbers found?

Subtract them and take the absolute value: distance = |a – b|.

Find the temperature difference between –5 °C and 7 °C.

|7 – (–5)| = |12| = 12 °C.

State the properties of real numbers with respect to addition and multiplication.

What are the closure and associative properties?

Sums and products of real numbers are real; grouping does not change the answer: (a + b) + c = a + (b + c), (ab)c = a(bc).

What are the identity properties?

$a + 0 = a$, so 0 is the additive identity; $a \times 1 = a$, so 1 is the multiplicative identity.

What are the inverse properties?

$a + (-a) = 0$ and $a \times 1/a = 1$ for $a \neq 0$.

What are the commutative and distributive properties?

$a + b = b + a$, $ab = ba$, and $a(b + c) = ab + ac$.

Explain the order properties of real numbers.

What is the trichotomy property?

For any a and b, exactly one of $a < b$, $a = b$, $a > b$ is true.

What is the transitive property?

If $a > b$ and $b > c$, then $a > c$; e.g., $8 > 5$ and $5 > 4$ give $8 > 4$.

What happens when we add or multiply?

Adding the same number keeps the sign; multiplying by a positive number keeps it, but multiplying by a negative number reverses it.

What about reciprocals?

For numbers of the same sign, $a < b$ means $1/a > 1/b$.

Solve the word problems on fractions and decimals.

12½ kg of sugar is packed in bags of 2½ kg. How many bags?

$25/2 \div 5/2 = 25/2 \times 2/5 = 5$ bags.

A tank holds 92.4 litres. How many 3.5 litre bottles can be filled?

$92.4 \div 3.5 = 26.4$, so 26 full bottles (with 1.4 litres left over).

A green ribbon is 3/4 of a 10 m blue ribbon. Find its length.

$3/4 \times 10 = 7.5$ m.

Find the total length of both ribbons.

$10 + 7.5 = 17.5$ m.

Multiple Choice Questions (MCQs)

Which of these is irrational? (A) 0.25 (B) √16 (C) √5 (D) 22/7
Correct answer: (C) √5. 5 is not a perfect square.

The set of integers is denoted by: (A) N (B) W (C) Z (D) Q
Correct answer: (C) Z. Z.

1/3 in decimal form is: (A) Terminating (B) Recurring (C) Irrational (D) Whole

Correct answer: (B) Recurring. 0.333....

Which number is not rational? (A) –7 (B) 0 (C) π (D) 3/8
Correct answer: (C) π. π is irrational.

|–25| equals: (A) –25 (B) 25 (C) 0 (D) 1/25
Correct answer: (B) 25. Distance from zero.

Distance between –6 and 3 is: (A) 3 (B) –9 (C) 9 (D) –3
Correct answer: (C) 9. |–6 – 3| = 9.

The additive identity is: (A) 1 (B) 0 (C) –1 (D) a
Correct answer: (B) 0. a + 0 = a.

The multiplicative inverse of 2/7 is: (A) –2/7 (B) 7/2 (C) –7/2 (D) 1
Correct answer: (B) 7/2. 2/7 × 7/2 = 1.

Which number has no multiplicative inverse? (A) 1 (B) –1 (C) 0 (D) 1/2
Correct answer: (C) 0. Nothing times 0 gives 1.

a + b = b + a is the ___ property. (A) Associative (B) Commutative (C) Distributive (D) Closure
Correct answer: (B) Commutative. Commutative.

3(x + 4) = 3x + 12 shows the ___ property. (A) Distributive (B) Identity (C) Inverse (D) Transitive
Correct answer: (A) Distributive. Distributive.

If a < b and c < 0, then: (A) ac < bc (B) ac > bc (C) ac = bc (D) a = b
Correct answer: (B) ac > bc. Multiplying by a negative reverses the sign.

If a > b and b > c then a > c is the ___ property. (A) Trichotomy (B) Transitive (C) Closure (D) Additive
Correct answer: (B) Transitive. Transitive.

7/5 lies between: (A) 0 and 1 (B) 1 and 2 (C) 2 and 3 (D) –1 and 0
Correct answer: (B) 1 and 2. 7/5 = 1.4.

Which decimal is terminating? (A) 0.666... (B) 0.142857... (C) 0.125 (D) 1.4142...
Correct answer: (C) 0.125. 0.125 = 1/8.

Quick Revision Summary

- N = {1, 2, 3, ...}; W adds 0; Z adds negatives; Q = p/q (q ≠ 0); Q′ = irrational; ℝ = Q ∪ Q′.

- Rational decimals terminate or recur; irrational decimals neither end nor repeat.
- π ≈ 22/7, but they are not equal.
- To show l/m on a number line, divide each unit into m parts.
- |x| = distance from 0; distance between a and b = |a – b|.
- Properties: closure, associative, commutative, identity (0 and 1), inverse (–a and 1/a), distributive.
- Order: trichotomy, transitive; multiply/divide by a negative → reverse the sign.

Exam Tips

- To test if a number is rational, try writing it as p/q; any terminating or recurring decimal works.
- Remember: square roots of non-perfect squares are irrational.
- In property questions, look for what changes: order (commutative), brackets (associative) or a bracket being opened (distributive).
- Whenever you multiply or divide an inequality by a negative number, flip the sign.
- Show the division step clearly in fraction word problems: 'how many pieces' means divide.

Note on Facts

When studying the transitive property, make sure the middle number is shared: the correct pattern is 8 > 5 and 5 > 4, so 8 > 4. An example in which the two statements do not share a middle number does not illustrate transitivity.

Unit 2: Estimation and Approximation

Unit 2 is about rounding numbers sensibly. In daily life exact values are often unnecessary or impossible to use: a price of Rs. 19.75 is paid as Rs. 20, and the height of K-2 (8611 m) may be quoted as about 8600 m. Rounding to a stated degree of accuracy gives a simpler, approximate value.

These notes cover significant figures, rounding whole numbers and decimals to significant figures (s.f.) or decimal places (d.p.), rounding negative numbers and fractions, finding the approximation error, and using approximation in real-life problems.

Learning Objectives

- Identify significant figures in whole numbers and decimals.
- Round whole numbers and decimals to a given number of significant figures or decimal places.
- Round negative integers and rational numbers.
- Estimate a calculation and find the approximation error.
- Apply rounding and approximation to real-life situations.

Key Concepts

2.1 Estimation and Significant Figures

Estimation means replacing an exact value with a nearby, simpler value to a required degree of accuracy.

Significant figures: the first non-zero digit from the left is the first significant figure. All non-zero digits are significant, and zeros between non-zero digits are significant (e.g., 4007 has 4 s.f.).

In a decimal, zeros before the first non-zero digit are not significant (0.0052 has 2 s.f.), while zeros after a non-zero digit in the decimal part are significant (2.50 has 3 s.f.).

Rounding to more significant figures gives a more accurate value: 343 m rounded to 2 s.f. (340) is closer than 1 s.f. (300).

2.2 Rounding Whole Numbers and Decimals

Rule: look at the digit just after the required place. If it is 5 or more, add 1 to the digit in the required place; if it is less than 5, leave that digit as it is. Then replace the remaining digits of a whole number by zeros, or drop them after a decimal point.

Example: 548735 → 500000 (1 s.f.), 549000 (3 s.f.), 548740 (5 s.f.).

Example: 87.5376 → 87.5 (1 d.p.), 87.54 (4 s.f.), 87.538 (3 d.p.). For 0.0785229, the first significant figure is 7, so 3 s.f. gives 0.0785.

s.f. stands for significant figures; d.p. stands for decimal places.

2.3 Rounding Negative Numbers and Rational Numbers

For a negative number, remember that the number nearer to 0 is greater. Rounding –234576 to 3 s.f. can go towards zero (–234000) or away from zero (–235000); the nearest value is –235000 because the next digit (5) is 5 or more.

Example: –48.3567 to 3 d.p. is –48.357 away from zero and –48.356 towards zero.

To round a fraction, first change it to a decimal, then round: 17/3 = 5.6666... ≈ 5.7 (2 s.f.); 235/28 = 8.392857... ≈ 8.39 (2 d.p.).

2.4 Approximation Error

To estimate a calculation, round each number to a simple value and work out the result; then calculate the accurate value and compare.

Approximation error (also called bias) shows how far the approximate value is from the accurate value. It is best found as the positive difference: |approximate value – accurate value|.

Example: 4.53 + 786 ≈ 5 + 800 = 805, while the accurate value is 790.53, so the error is 805 – 790.53 = 14.47.

Example: (1.45 × 560) ÷ 23.5 ≈ (1 × 600) ÷ 24 = 25, while the accurate value is about 34.55, so the error is 34.55 – 25 = 9.55.

2.5 Real-Life Approximation

Large measurements are usually given to a suitable number of significant figures: the least Earth–Moon distance of 363104 km is 363000 km to 3 s.f. and 363100 km to 5 s.f.

The symbol ≈ means 'is approximately equal to'.

Important Definitions

Estimation
Replacing an exact value with a nearby simpler value to a required accuracy.

Rounding
Writing a number to a required number of significant figures or decimal places.

Significant figure
A digit that contributes to the accuracy of a number, starting from the first non-zero digit.

Decimal place
A digit position after the decimal point.

Degree of accuracy
The number of s.f. or d.p. to which a value is rounded.

Approximate value

The rounded or estimated value of a quantity.

Accurate value
The exact value obtained by full calculation.

Approximation error
The difference between the approximate and accurate values; also called bias.

Rounding towards zero
Rounding a negative number to the value nearer zero.

Rounding away from zero
Rounding a negative number to the value farther from zero.

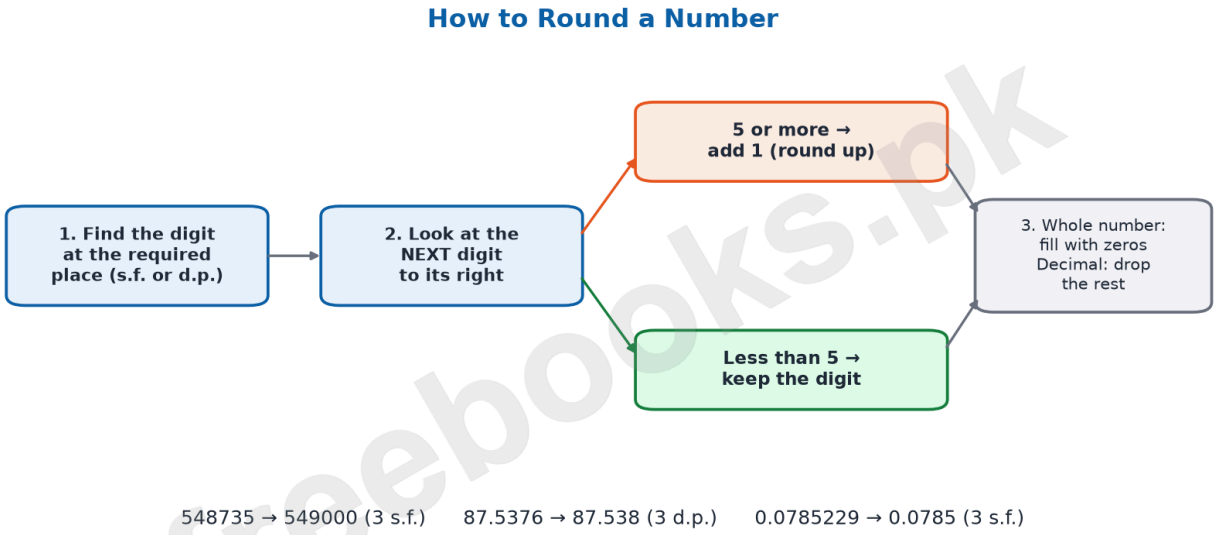
≈
The symbol for 'approximately equal to'.

Key Facts at a Glance

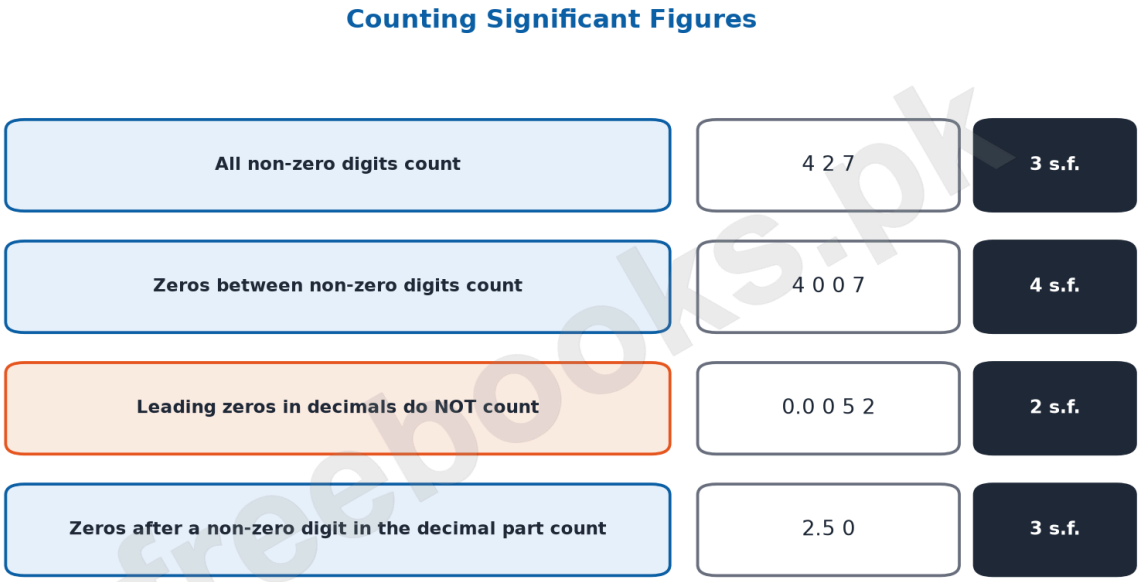
Number	Rounded
548735 (1 s.f.)	500000
548735 (3 s.f.)	549000
87.5376 (1 d.p.)	87.5
87.5376 (3 d.p.)	87.538
0.0785229 (3 s.f.)	0.0785
17/3 (2 s.f.)	5.7
235/28 (2 d.p.)	8.39
363104 km (3 s.f.)	363000 km
Significant figures in 0.0052	2
Significant figures in 4007	4
Significant figures in 2.50	3
Error: 805 vs 790.53	14.47

Diagrams

Rounding rule: A flow chart of the steps for rounding a number to a given degree of accuracy.

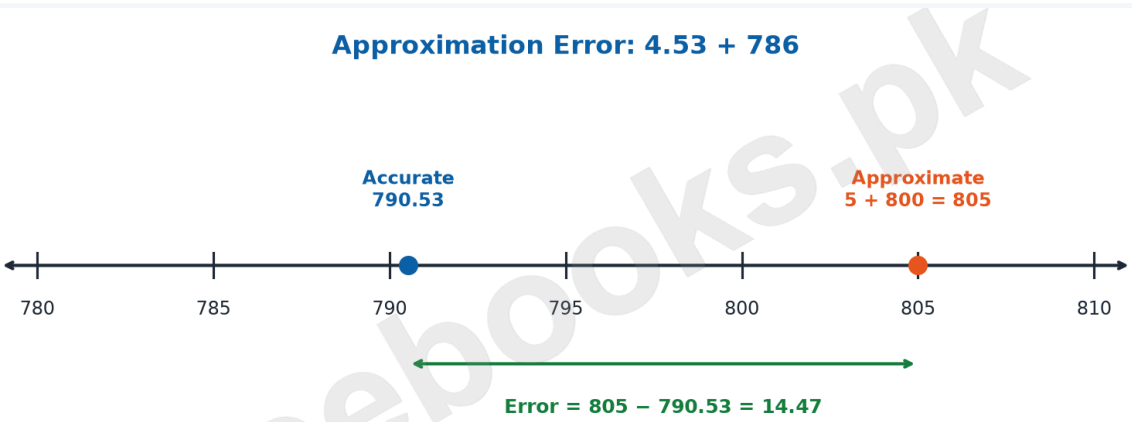


Counting significant figures: A chart showing which digits and zeros are significant, with examples.



Approximation error: A number line comparing an approximate value with the accurate value.

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Short Questions and Answers

What is estimation?
Replacing an exact value with a simpler value close to it, to a required degree of accuracy.

How many significant figures are in 30.07?
4 — the zeros between non-zero digits count.

How many significant figures are in 0.00610?
3 — leading zeros do not count, but the final zero after 1 does.

Round 7296 to 2 s.f.
7300 (the next digit 9 is more than 5, so 72 becomes 73).

Round 3.14159 to 2 d.p. and to 4 s.f.
3.14 (2 d.p.) and 3.142 (4 s.f.).

Round 0.004862 to 2 s.f.
0.0049.

Why is 2 s.f. more accurate than 1 s.f.?
It keeps more digits of the original number, so the rounded value is closer to the exact value.

Which is greater, –12 or –15?
–12, because it is nearer to zero.

Round –6.85 to 1 d.p. away from zero.
–6.9.

Round 22/7 to 3 d.p.
22/7 = 3.142857... ≈ 3.143.

What is approximation error?
The difference between the approximate value and the accurate value of a calculation.

Estimate 49.6 × 2.03 and find the error.
Estimate 50 × 2 = 100; accurate 100.688; error ≈ 0.688.

What does the symbol ≈ mean?
Approximately equal to.

Round Rs. 249.75 to the nearest rupee.
Rs. 250.

Round 8848.86 m (height of Mount Everest) to 3 s.f.
8850 m.

Long Questions and Answers

Explain significant figures with rules and examples.

What is the first significant figure?

The first non-zero digit from the left; in 0.0372 it is 3.

Which zeros are significant?

Zeros between non-zero digits (e.g., 5003) and zeros after a non-zero digit in the decimal part (e.g., 1.20).

Which zeros are not significant?

Leading zeros in a decimal, such as the zeros in 0.006.

Why does the number of s.f. matter?

More significant figures means a more accurate rounded value.

Describe the method of rounding with examples.

What is the rounding rule?

Look at the next digit after the required place: 5 or more → round up; less than 5 → keep the digit.

Round 64,527 to 1, 2 and 3 s.f.

60,000 (1 s.f.), 65,000 (2 s.f.), 64,500 (3 s.f.).

Round 12.4683 to 1 d.p. and 3 d.p.

12.5 (1 d.p.) and 12.468 (3 d.p.).

Round 0.05096 to 2 s.f.

0.051.

Explain how negative numbers and fractions are rounded.

What should be remembered about negative numbers?

The number nearer zero is greater, so –3 > –8.

Round –5,672 to 2 s.f. towards and away from zero.

Towards zero –5,600; away from zero –5,700 (also the nearest value).

How is a fraction rounded?

Divide to get a decimal first, then apply the rounding rule.

Round 5/7 to 3 d.p.

5/7 = 0.714285... ≈ 0.714.

Explain approximation error with a worked example.

What are the steps?

Round each number, calculate the approximate answer, calculate the accurate answer, then find the difference.

Estimate 6.8 × 19.7.

7 × 20 = 140.

Find the accurate value.

6.8 × 19.7 = 133.96.

Find the error.

140 – 133.96 = 6.04.

Give real-life uses of rounding and approximation.

Why do shopkeepers round prices?

Small coins may not be available, so amounts like Rs. 99.60 are paid as Rs. 100.

How are large distances reported?

Rounded to a few significant figures, e.g., 363104 km ≈ 363000 km.

Why is estimation useful before calculating?

It gives a quick check on whether the exact answer is reasonable.

Round the Earth's diameter 12742 km to 2 s.f.

13000 km.

Round the following and state the degree of accuracy.

Round 3,85,642 to 3 s.f.

3,86,000.

Round 0.0004567 to 2 s.f.

0.00046.

Round 49.996 to 2 d.p.

50.00.

Round –0.8452 to 2 s.f. away from zero.

–0.85.

Multiple Choice Questions (MCQs)

The first significant figure of 0.0607 is: (A) 0 (B) 6 (C) 7 (D) 1
Correct answer: (B) 6. First non-zero digit.

How many s.f. in 5002? (A) 1 (B) 2 (C) 3 (D) 4
Correct answer: (D) 4. Middle zeros count.

How many s.f. in 0.0040? (A) 1 (B) 2 (C) 3 (D) 4
Correct answer: (B) 2. 4 and the final 0.

4763 rounded to 2 s.f. is: (A) 4700 (B) 4800 (C) 4760 (D) 5000
Correct answer: (B) 4800. Next digit 6 → round up.

2.785 rounded to 2 d.p. is: (A) 2.78 (B) 2.79 (C) 2.8 (D) 2.70
Correct answer: (B) 2.79. Next digit 5 → round up.

0.0289357 to 3 s.f. is: (A) 0.029 (B) 0.0289 (C) 0.02894 (D) 0.03
Correct answer: (B) 0.0289. 2, 8, 9.

d.p. stands for: (A) Digit place (B) Decimal place (C) Decimal point (D) Division part
Correct answer: (B) Decimal place. Decimal place.

Which is greater? (A) –7 (B) –2 (C) –10 (D) –5
Correct answer: (B) –2. Nearest to zero.

–45325 to 3 s.f. away from zero is: (A) –45300 (B) –45400 (C) –45000 (D) –46000
Correct answer: (B) –45400. Away from zero.

23/3 to 2 d.p. is: (A) 7.66 (B) 7.67 (C) 7.6 (D) 7.7
Correct answer: (B) 7.67. 7.666... → 7.67.

The symbol for approximately equal is: (A) = (B) ≠ (C) ≈ (D) ≤
Correct answer: (C) ≈. ≈.

Approximation error is also called: (A) Bias (B) Mean (C) Range (D) Ratio

Correct answer: (A) Bias. Bias.

Estimate of 19.8 × 5.1 is: (A) 90 (B) 100 (C) 110 (D) 120
Correct answer: (B) 100. 20 × 5.

More significant figures give a value that is: (A) Less accurate (B) More accurate (C) Negative (D) Unchanged
Correct answer: (B) More accurate. Closer to exact.

12742 to 3 s.f. is: (A) 12700 (B) 12800 (C) 12740 (D) 13000
Correct answer: (A) 12700. Next digit 4 → keep.

Quick Revision Summary

- Estimation/rounding gives a simpler value to a required accuracy (s.f. or d.p.).
- First s.f. = first non-zero digit; middle zeros count; leading decimal zeros do not.
- Rule: next digit ≥ 5 → round up; < 5 → keep.
- Negative numbers: nearer zero is greater; round towards or away from zero as asked.
- Fractions: convert to decimal first, then round.
- Approximation error = difference between approximate and accurate values (bias).

Exam Tips

- Underline the digit at the required place and circle the next digit before rounding.
- Keep zeros as place holders in whole numbers (548735 → 549000, not 549).
- For decimals rounded to d.p., keep a final zero if needed (49.996 → 50.00 to 2 d.p.).
- In error questions, write the approximate value, accurate value and error on separate lines.
- Read whether the question asks to round 'towards' or 'away from' zero for negative numbers.

Note on Facts

The textbook's two worked examples subtract in different orders (approximate – accurate in one, accurate – approximate in the other). Approximation error is usually stated as a positive amount, so take the positive difference: |approximate value – accurate value|.

For negative numbers the textbook gives both a 'towards zero' and an 'away from zero' answer. Under the ordinary rounding rule (next digit 5 or more rounds the size up), –234576 to 3 s.f. is –235000; the 'towards zero' value –234000 simply drops the extra digits. Follow the direction asked in the question.

Unit 3: Square Roots and Cube Roots

Unit 3 deals with squares, square roots, cubes and cube roots. Squaring a number multiplies it by itself ($x \times x = x^2$), and finding the square root reverses this. Cubing multiplies a number by itself three times (x^3), and the cube root reverses that.

These notes explain perfect squares and their patterns, finding square roots of whole numbers, fractions and decimals by prime factorization and by the division method, approximating square roots of non-perfect squares, the rule for the number of digits in a square root, cubes and cube roots by prime factorization, and real-life problems.

Learning Objectives

- Recognise perfect squares and perfect cubes and their patterns.
- Find square roots by prime factorization and by the division method.
- Find square roots of fractions and decimals.
- Find the square root of a non-perfect square up to 2 decimal places.
- Use the rule for the number of digits in a square root.
- Find cube roots of perfect cubes by prime factorization and solve word problems.

Key Concepts

3.1 Squares and Perfect Squares

The square of a number is the number multiplied by itself: $13^2 = 169$. A perfect square is the square of a whole number, such as 1, 4, 9, 16, 25, ...

Patterns: a perfect square always ends in 0, 1, 4, 5, 6 or 9, never in 2, 3, 7 or 8. The square of an even number is even and of an odd number is odd. The sum of the first n odd numbers is n^2 ($1 + 3 + 5 + 7 = 16 = 4^2$).

The square of a fraction or decimal squares both parts: $(3/5)^2 = 9/25$ and $(0.4)^2 = 0.16$.

3.2 Square Roots by Prime Factorization

The square root is the inverse of squaring: $\sqrt{169} = 13$ because $13^2 = 169$. The symbol $\sqrt{}$ is called the radical sign.

Method: write the number as a product of prime factors, pair equal factors, and take one factor from each pair. Example: $1764 = 2 \times 2 \times 3 \times 3 \times 7 \times 7$, so $\sqrt{1764} = 2 \times 3 \times 7 = 42$. If a factor is left without a pair, the number is not a perfect square.

To make a number a perfect square, multiply (or divide) by the unpaired factor: $72 = 2^3 \times 3^2$, so $72 \times 2 = 144 = 12^2$.

Fractions: $\sqrt{(a/b)} = \sqrt{a} / \sqrt{b}$, e.g., $\sqrt{(144/225)} = 12/15 = 4/5$. Decimals: change to a fraction, e.g., $\sqrt{0.0081} = \sqrt{(81/10000)} = 9/100 = 0.09$.

3.3 Square Roots by the Division Method

Pair the digits from the right (and from the decimal point for decimals). Find the largest number whose square is not more than the first pair, subtract, bring down the next pair, double the root found so far as a new trial divisor, and continue.

Example: $\sqrt{529}$ — the first group is 5, the largest square ≤ 5 is 4 (2^2); remainder 1, bring down 29 to make 129; double 2 to get 4_, and 43 × 3 = 129, so $\sqrt{529} = 23$.

For a non-perfect square, add pairs of zeros after the decimal point and continue to one more place than needed, then round: $\sqrt{7} \approx 2.645... \approx 2.65$; $\sqrt{10} \approx 3.16$.

3.4 Number of Digits in a Square Root

If a perfect square has n digits, its square root has n/2 digits when n is even and (n + 1)/2 digits when n is odd.

Examples: 4096 has 4 digits, so $\sqrt{4096}$ has 2 digits (64). 49729 has 5 digits, so its root has 3 digits (223).

3.5 Cubes and Cube Roots

The cube of a number is the number used three times as a factor: $5^3 = 125$. A perfect cube is the cube of a whole number: 1, 8, 27, 64, 125, 216, ...

The cube of an even number is even ($6^3 = 216$) and of an odd number is odd ($7^3 = 343$). The cube of a negative number is negative.

The cube root $\sqrt[3]{x}$ is the number whose cube is x. By prime factorization, group equal factors in threes and take one from each group: $13824 = 2^9 \times 3^3$ so $\sqrt[3]{13824} = 2^3 \times 3 = 24$.

Fractions and decimals: $\sqrt[3]{(27/125)} = 3/5$; $\sqrt[3]{0.008} = \sqrt[3]{(8/1000)} = 2/10 = 0.2$.

3.6 Real-Life Problems

Square roots give the side of a square from its area: a square field of 1296 m^2 has side $\sqrt{1296} = 36$ m (perimeter 144 m).

Cube roots give the edge of a cube from its volume: a cubical box of 343 cm^3 has edge $\sqrt[3]{343} = 7$ cm.

Square roots also appear when arranging items in equal rows and columns, such as 625 plants planted in a square (25 rows of 25).

Important Definitions

Square
A number multiplied by itself, written x^2.

Perfect square
The square of a whole number, such as 36 or 144.

Square root
A number whose square equals the given number, written $\sqrt{x}$.

Radical sign
The symbol $\sqrt{}$ used for roots.

Prime factorization
Writing a number as a product of prime numbers.

Division method
A step-by-step long division process for finding square roots.

Non-perfect square
A number whose square root is not a whole number, e.g., 7.

Cube
A number used three times as a factor, written x^3.

Perfect cube
The cube of a whole number, such as 64 or 343.

Cube root
A number whose cube equals the given number, written $\sqrt[3]{x}$.

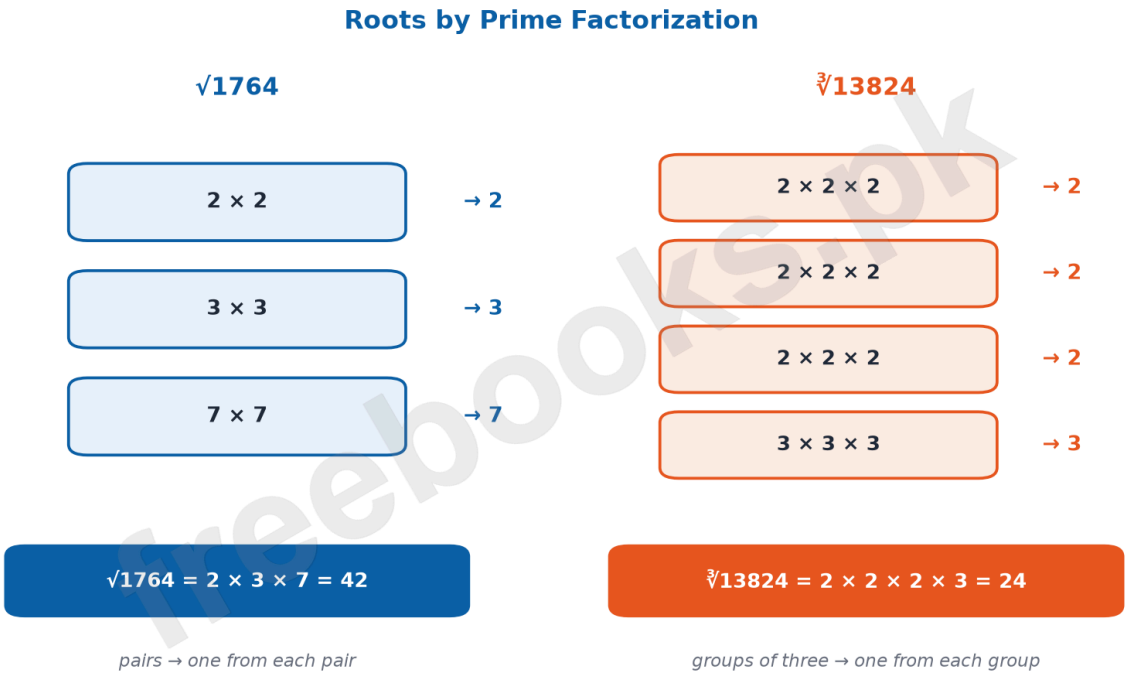
Key Facts at a Glance

Item	Value / rule
Squares end in	0, 1, 4, 5, 6 or 9
1 + 3 + 5 + ... (n odd numbers)	n^2
$\sqrt{1764}$	42
$\sqrt{529}$	23
$\sqrt{(144/225)}$	4/5
$\sqrt{0.0081}$	0.09
$\sqrt{7}$ (2 d.p.)	2.65
Digits in $\sqrt{}$ of n-digit square	n/2 (even n), (n+1)/2 (odd n)
Cubes of 1 to 6	1, 8, 27, 64, 125, 216
$\sqrt[3]{13824}$	24

∛(27/125)	3/5
∛0.008	0.2

Diagrams

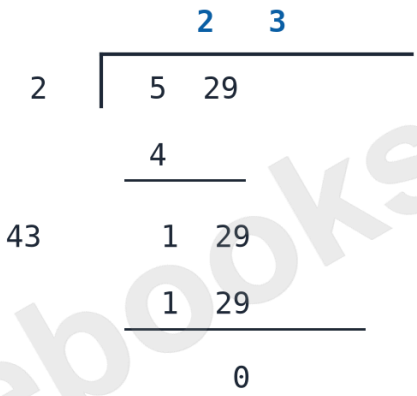
Square root by prime factorization: Pairing prime factors of 1764 to find its square root, and grouping in threes for a cube root.



Division method for √529: The long division layout used to find the square root of 529.

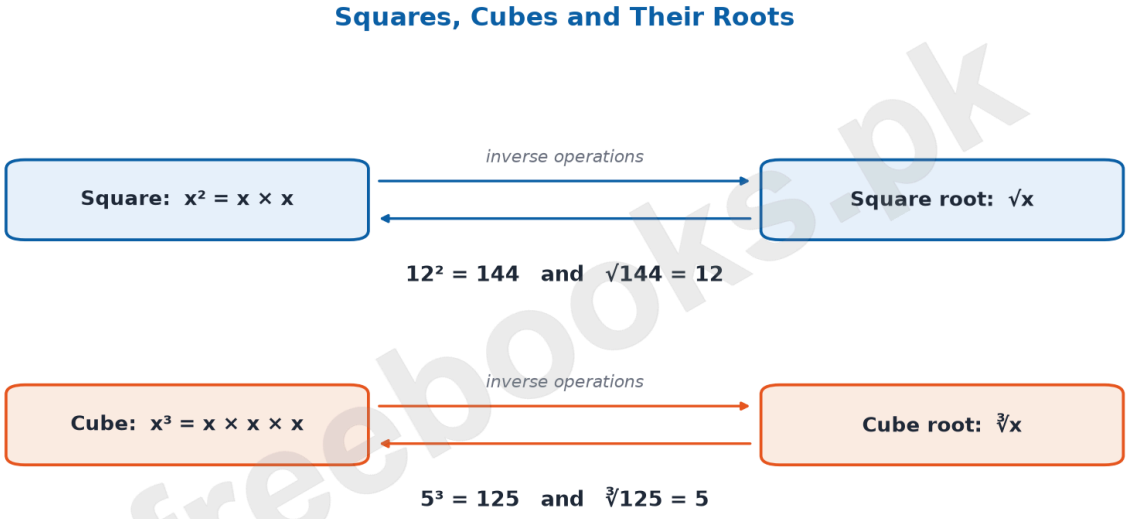
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Division Method: √529 = 23



1. Pair digits: 5 | 29
2. $2^2 = 4 \leq 5 \rightarrow$ write 2
3. $5 - 4 = 1$; bring down 29 → 129
4. Double 2 → 4_ ; $43 \times 3 = 129$
5. **Remainder 0, so √529 = 23**

Squares, cubes and their roots: A chart showing squaring and cubing with their inverse operations, with examples.



Short Questions and Answers

Can 1078 be a perfect square? Why?
No. Perfect squares never end in 8.

Write 1 + 3 + 5 + 7 + 9 + 11 as a square.
These are the first 6 odd numbers, so the sum is $6^2 = 36$.

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Find $\sqrt{2025}$ by prime factorization.
$2025 = 3^4 \times 5^2$, so $\sqrt{2025} = 3^2 \times 5 = 45$.

By what smallest number should 180 be multiplied to get a perfect square?
$180 = 2^2 \times 3^2 \times 5$, so multiply by 5 to get $900 = 30^2$.

Find $\sqrt{(49/196)}$.
$7/14 = 1/2$.

Find $\sqrt{2.25}$.
$\sqrt{(225/100)} = 15/10 = 1.5$.

How many digits will $\sqrt{14641}$ have?
14641 has 5 digits (odd), so the root has $(5 + 1)/2 = 3$ digits; $\sqrt{14641} = 121$.

Find $\sqrt{10}$ to 2 decimal places.
$\sqrt{10} \approx 3.162... \approx 3.16$.

Is the cube of 9 odd or even?
Odd, because 9 is odd ($9^3 = 729$).

Find $\sqrt[3]{2744}$.
$2744 = 2^3 \times 7^3$, so $\sqrt[3]{2744} = 2 \times 7 = 14$.

What is the smallest number by which 500 must be multiplied to get a perfect cube?
$500 = 2^2 \times 5^3$, so multiply by 2 to get $1000 = 10^3$.

Find $\sqrt[3]{(-64)}$.
-4, because $(-4)^3 = -64$.

Find $\sqrt[3]{0.027}$.
$\sqrt[3]{(27/1000)} = 3/10 = 0.3$.

A square garden has area 625 m². Find its side.
$\sqrt{625} = 25$ m.

A cube has volume 3375 cm³. Find its edge.
$\sqrt[3]{3375} = 15$ cm.

Long Questions and Answers

Explain how to find a square root by prime factorization.

What is the first step?

Divide the number repeatedly by primes to write it as a product of prime factors.

How are the factors used?

Group equal factors in pairs and take one factor from each pair.

Find √3136.

$3136 = 2^6 \times 7^2$, so $\sqrt{3136} = 2^3 \times 7 = 56$.

How can you tell a number is not a perfect square?

At least one prime factor is left without a partner, e.g., $50 = 2 \times 5^2$, where 2 is unpaired.

Explain the division method with an example.

How are digits grouped?

Into pairs from the right for whole numbers, and from the decimal point in both directions for decimals.

What is done with the first pair?

Find the largest square not more than it, write its root above, subtract, and bring down the next pair.

How is the next digit found?

Double the root so far, use it as the start of a new divisor, and choose the digit that makes divisor × digit as large as possible without exceeding the remainder.

Find √1369.

Groups 13|69: $3^2 = 9$, remainder 4, bring down 69 → 469; $67 \times 7 = 469$, so $\sqrt{1369} = 37$.

Find the square roots of fractions and decimals.

What rule is used for fractions?

Take the square root of the numerator and denominator separately: $\sqrt{(a/b)} = \sqrt{a}/\sqrt{b}$.

Find √(81/256).

9/16.

How are decimals handled?

Write the decimal as a fraction with a square denominator such as 100 or 10000.

Find √1.44 and √0.0049.

$\sqrt{(144/100)} = 1.2$; $\sqrt{(49/10000)} = 0.07$.

Explain cubes and cube roots with properties.

What is a perfect cube?

A number obtained by cubing a whole number, e.g., $8 = 2^3$ and $1000 = 10^3$.

What are some properties of cubes?

Cubes of even numbers are even, of odd numbers are odd, and of negative numbers are negative.

How is a cube root found by prime factorization?

Group equal prime factors in threes and take one factor from each group.

Find $\sqrt[3]{17576}$.

$17576 = 2^3 \times 13^3$, so $\sqrt[3]{17576} = 26$.

Solve real-life problems using square and cube roots.

A square hall floor is 784 m². Find its side and perimeter.

Side $\sqrt{784} = 28$ m; perimeter $4 \times 28 = 112$ m.

2304 soldiers stand in a square formation. How many are in each row?

$\sqrt{2304} = 48$ soldiers.

A cubical tank holds 1728 litres (1 litre = 1000 cm³). Find its inner edge.

Volume 1,728,000 cm³; $\sqrt[3]{1{,}728{,}000} = 120$ cm.

A cube of edge 4 cm is melted into small cubes of edge 1 cm. How many cubes?

$4^3 = 64$ small cubes.

Explain how to find the square root of a non-perfect square.

Why is the answer approximate?

The root of a non-perfect square is irrational, so its decimal never ends.

How is the division method extended?

Pairs of zeros are added after the decimal point and the process is continued.

How many places should be found?

One more than required, so the answer can be rounded correctly.

Find $\sqrt{3}$ to 2 decimal places.

$\sqrt{3} = 1.732... \approx 1.73$.

Multiple Choice Questions (MCQs)

Which can be a perfect square? (A) 243 (B) 567 (C) 961 (D) 1022
Correct answer: (C) 961. $961 = 31^2$.

The sum of the first 8 odd numbers is: (A) 56 (B) 64 (C) 72 (D) 81
Correct answer: (B) 64. $8^2 = 64$.

$\sqrt{1764}$ = (A) 38 (B) 42 (C) 44 (D) 48

Correct answer: (B) 42. 2 × 3 × 7.

√0.09 = (A) 0.03 (B) 0.3 (C) 0.9 (D) 3
Correct answer: (B) 0.3. $0.3^2 = 0.09$.

√(16/49) = (A) 4/7 (B) 8/7 (C) 16/7 (D) 4/49
Correct answer: (A) 4/7. 4/7.

A 6-digit perfect square has a square root of: (A) 2 digits (B) 3 digits (C) 4 digits (D) 6 digits
Correct answer: (B) 3 digits. 6/2 = 3.

√7 to 2 d.p. is about: (A) 2.45 (B) 2.65 (C) 2.75 (D) 2.85
Correct answer: (B) 2.65. 2.6457...

The radical sign is: (A) π (B) √ (C) ∛ only (D) ≈
Correct answer: (B) √. √.

5^3 = (A) 15 (B) 25 (C) 125 (D) 625
Correct answer: (C) 125. 5 × 5 × 5.

∛216 = (A) 4 (B) 6 (C) 8 (D) 36
Correct answer: (B) 6. $6^3 = 216$.

The cube of an odd number is: (A) Even (B) Odd (C) Zero (D) Negative
Correct answer: (B) Odd. Odd × odd × odd.

∛(–27) = (A) 3 (B) –3 (C) 9 (D) –9
Correct answer: (B) –3. $(-3)^3 = -27$.

∛0.125 = (A) 0.5 (B) 0.05 (C) 0.25 (D) 5
Correct answer: (A) 0.5. $0.5^3 = 0.125$.

The smallest number to multiply 72 by to get a perfect square is: (A) 2 (B) 3 (C) 4 (D) 6
Correct answer: (A) 2. $72 = 2^3 \times 3^2$.

A cube of volume 512 cm^3 has edge: (A) 6 cm (B) 7 cm (C) 8 cm (D) 9 cm
Correct answer: (C) 8 cm. $8^3 = 512$.

Quick Revision Summary

- $x^2 = x \times x$; √ is the inverse of squaring. Squares end in 0, 1, 4, 5, 6, 9.
- Prime factorization: pair factors for √, group in threes for ∛.
- Division method: pair digits, double the root, find the next digit.

- $\sqrt{(a/b)} = \sqrt{a}/\sqrt{b}$; change decimals to fractions with square denominators.
- Non-perfect squares: add pairs of zeros and round to the required places.
- Digits in $\sqrt{}$: n/2 if n even, (n+1)/2 if n odd.
- $x^3 = x \times x \times x$; cubes keep the sign and parity of the number.

Exam Tips

- Learn squares up to 25^2 and cubes up to 12^3; they speed up every question.
- Use the last-digit rule to rule out non-squares quickly.
- Show the full prime factorization; marks are given for the working.
- In the division method, keep columns neat and always double the root found so far.
- Check answers by squaring or cubing back.

Unit 4: Sets

Unit 4 is about sets: well-defined collections of distinct objects called elements or members. Sets help us sort and compare groups of numbers or things, and they are the basis of many later topics such as probability and functions.

These notes cover important number sets, the three ways of writing a set, equal and equivalent sets, subsets and power sets, the operations of union, intersection, difference and complement, the commutative, associative and distributive laws, De Morgan's laws, and Venn diagrams for overlapping, disjoint and subset cases.

Learning Objectives

- Write a set in tabular, descriptive and set-builder form.
- Distinguish equal and equivalent sets.
- List all subsets and the power set of a set with up to four elements.
- Find the union, intersection, difference and complement of sets.
- Verify the commutative, associative and distributive laws and De Morgan's laws.
- Represent set operations on Venn diagrams and solve real-life problems.

Key Concepts

4.1 Sets and Their Notation

A set is a well-defined collection of distinct objects, written with capital letters and braces: A = {2, 4, 6}. We write $4 \in A$ ('4 belongs to A') and $5 \notin A$.

Important sets: N (natural numbers), W (whole numbers), Z (integers), E (even numbers), O (odd numbers), P (prime numbers), Q (rational numbers). The empty set ∅ or { } has no element.

Three forms: tabular (roster) form lists elements, e.g., A = {1, 3, 5, 7}; descriptive form uses words, e.g., 'A is the set of odd numbers less than 8'; set-builder form states the rule, e.g., $A = \{x \mid x \in O, x < 8\}$.

4.2 Equal and Equivalent Sets

Equal sets have exactly the same elements (order does not matter): {1, 2, 3} = {3, 1, 2}.

Equivalent sets have the same number of elements, so their elements can be matched one-to-one: {a, b, c} ↔ {5, 6, 7}. Equal sets are always equivalent, but equivalent sets need not be equal.

4.3 Subsets and Power Set

A is a subset of B ($A \subseteq B$) if every element of A is also in B. A proper subset ($A \subset B$) is a subset that is not equal to B. The empty set is a subset of every set, and every set is a subset of itself.

The power set P(A) is the set of all subsets of A. If A has n elements, P(A) has 2^n elements. For A = {a, b}: P(A) = {∅, {a}, {b}, {a, b}} (4 elements).

Elements of P(A) are sets: {a} $\in$ P(A), but a $\notin$ P(A). P(∅) = {∅} has one element, so it is not empty.

4.4 Operations on Sets

Union A ∪ B: all elements in A or B (or both). Intersection A ∩ B: elements common to both. Difference A – B: elements in A but not in B.

The universal set U contains all elements under discussion. The complement A′ = U – A contains the elements of U not in A.

Example: U = {1, ..., 10}, A = {1, 2, 3, 4, 5}, B = {4, 5, 6, 7}. A ∪ B = {1, ..., 7}, A ∩ B = {4, 5}, A – B = {1, 2, 3}, B – A = {6, 7}, A′ = {6, 7, 8, 9, 10}.

Disjoint sets have no common element (A ∩ B = ∅). Overlapping sets share some elements.

4.5 Laws of Union and Intersection

Commutative: A ∪ B = B ∪ A and A ∩ B = B ∩ A. Associative: (A ∪ B) ∪ C = A ∪ (B ∪ C) and (A ∩ B) ∩ C = A ∩ (B ∩ C).

Distributive: A ∪ (B ∩ C) = (A ∪ B) ∩ (A ∪ C) and A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∩ C).

De Morgan's laws: (A ∪ B)′ = A′ ∩ B′ and (A ∩ B)′ = A′ ∪ B′. With the example sets above, (A ∪ B)′ = {8, 9, 10} = A′ ∩ B′.

4.6 Venn Diagrams and Real Life

In a Venn diagram, the universal set is a rectangle and sets are circles inside it; the required region is shaded. Overlapping circles show common elements, separate circles show disjoint sets, and a circle inside another shows a subset.

Real-life use: if 25 students like cricket, 18 like hockey and 10 like both, then the number who like at least one game is n(C ∪ H) = 25 + 18 – 10 = 33.

Important Definitions

Set
A well-defined collection of distinct objects.

Element
An object that belongs to a set.

Empty set
A set with no elements, written ∅ or { }.

Tabular form
Writing a set by listing its elements in braces.

Set-builder form

Writing a set by stating the common property of its elements.

Equal sets
Sets having exactly the same elements.

Equivalent sets
Sets having the same number of elements.

Subset
A set all of whose elements belong to another set.

Proper subset
A subset that is not equal to the whole set.

Power set
The set of all subsets of a set.

Universal set
The set containing all elements under consideration.

Union
The set of elements in A or B or both.

Intersection
The set of elements common to A and B.

Difference A – B
Elements of A that are not in B.

Complement
Elements of the universal set not in the given set.

Disjoint sets
Sets with no common element.

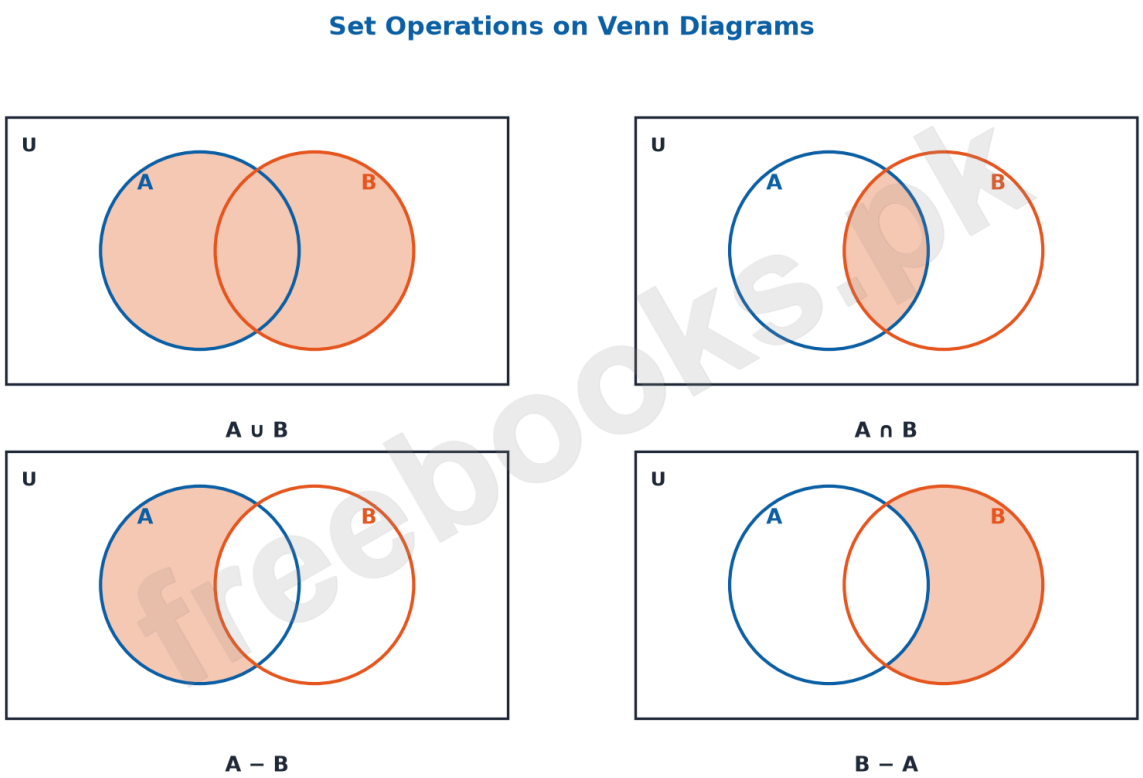
Key Facts at a Glance

Symbol / rule	Meaning
$\in$ / $\notin$	Belongs to / does not belong to
$\varnothing$	Empty set
$A \subseteq B$	A is a subset of B

n(P(A))	2^n, if A has n elements
A ∪ B	Union
A ∩ B	Intersection
A – B	Difference
A′	Complement = U – A
A ∩ B = ∅	A and B are disjoint
(A ∪ B)′	A′ ∩ B′ (De Morgan)
(A ∩ B)′	A′ ∪ B′ (De Morgan)
n(A ∪ B)	n(A) + n(B) – n(A ∩ B)

Diagrams

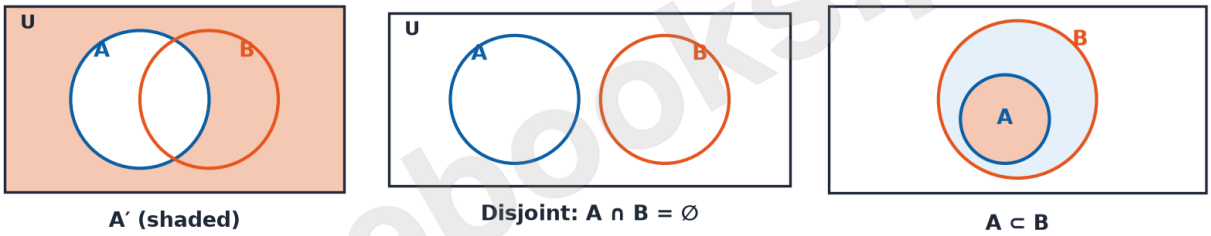
Union, intersection and difference: Venn diagrams with shaded regions for A ∪ B, A ∩ B, A – B and B – A.



Complement and disjoint sets: Venn diagrams for A′, disjoint sets and a subset inside a set.

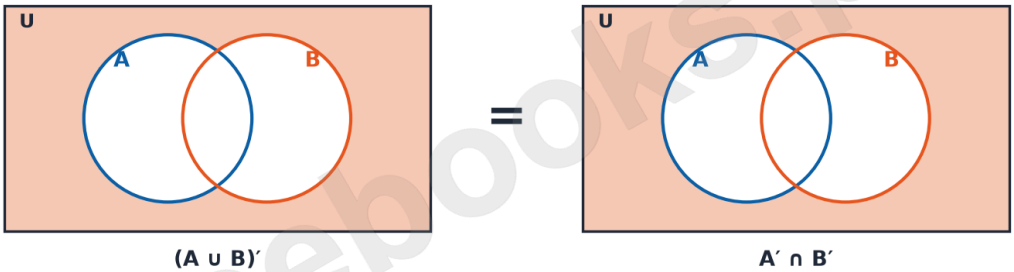
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Complement, Disjoint Sets and Subsets



De Morgan's law: Venn diagrams showing that (A ∪ B)′ and A′ ∩ B′ are the same region.

De Morgan's Law: (A ∪ B)′ = A′ ∩ B′



The shaded region lies outside both A and B in each diagram.

Short Questions and Answers

Write the set of prime numbers less than 15 in tabular form.
{2, 3, 5, 7, 11, 13}.

Write {5, 10, 15, 20} in set-builder form.
{x | x is a multiple of 5, x ≤ 20, x ∈ N}.

Are {1, 2, 3} and {a, b, c} equal or equivalent?
Equivalent (same number of elements), not equal.

Is ∅ a subset of {4, 5}?
Yes. The empty set is a subset of every set.

Write all subsets of {x, y}.
∅, {x}, {y}, {x, y}.

How many elements are in P(A) if A has 4 elements?

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$2^4 = 16$.

If A = {2, 4, 6, 8} and B = {6, 8, 10}, find A ∪ B.
{2, 4, 6, 8, 10}.

For the same sets, find A ∩ B.
{6, 8}.

Find A – B and B – A for the same sets.
A – B = {2, 4}; B – A = {10}.

If U = {1, 2, ..., 8} and A = {1, 3, 5, 7}, find A′.
{2, 4, 6, 8}.

When are two sets called disjoint?
When they have no common element, i.e., A ∩ B = ∅.

Write De Morgan's laws.
(A ∪ B)′ = A′ ∩ B′ and (A ∩ B)′ = A′ ∪ B′.

Is {a} ∈ P({a, b})? Is a ∈ P({a, b})?
{a} ∈ P({a, b}) is true; a ∈ P({a, b}) is false because elements of a power set are sets.

If A ⊆ B, what is A ∩ B?
A ∩ B = A.

30 students play football, 20 play hockey and 8 play both. How many play at least one?
30 + 20 – 8 = 42.

Long Questions and Answers

Explain the three ways of writing a set with examples.

What is tabular form?

Listing all elements in braces, separated by commas, e.g., V = {a, e, i, o, u}.

What is descriptive form?

Describing the set in words, e.g., 'V is the set of vowels of the English alphabet'.

What is set-builder form?

Writing the rule the elements satisfy, e.g., V = {x | x is a vowel of the English alphabet}.

Write the even numbers between 1 and 11 in all three forms.

{2, 4, 6, 8, 10}; the set of even numbers between 1 and 11; {x | x ∈ E, 1 < x < 11}.

Explain subsets and the power set.

When is A a subset of B?

When every element of A is also an element of B, written $A \subseteq B$.

What is a proper subset?

A subset of B that is not equal to B, written $A \subset B$.

What is the power set?

The set of all subsets of A; it has 2^n elements when A has n elements.

Write P(A) for A = {1, 2, 3}.

{∅, {1}, {2}, {3}, {1, 2}, {1, 3}, {2, 3}, {1, 2, 3}} — 8 elements.

Explain the operations on sets with an example.

Take U = {1, ..., 12}, A = {2, 4, 6, 8, 10, 12}, B = {3, 6, 9, 12}. Find A ∪ B.

{2, 3, 4, 6, 8, 9, 10, 12}.

Find A ∩ B.

{6, 12}.

Find A – B and B – A.

A – B = {2, 4, 8, 10}; B – A = {3, 9}.

Find A′ and B′.

A′ = {1, 3, 5, 7, 9, 11}; B′ = {1, 2, 4, 5, 7, 8, 10, 11}.

Verify De Morgan's laws for the sets above.

Find (A ∪ B)′.

U – {2, 3, 4, 6, 8, 9, 10, 12} = {1, 5, 7, 11}.

Find A′ ∩ B′.

{1, 3, 5, 7, 9, 11} ∩ {1, 2, 4, 5, 7, 8, 10, 11} = {1, 5, 7, 11}; so (A ∪ B)′ = A′ ∩ B′.

Find (A ∩ B)′.

U – {6, 12} = {1, 2, 3, 4, 5, 7, 8, 9, 10, 11}.

Find A′ ∪ B′.

{1, 2, 3, 4, 5, 7, 8, 9, 10, 11}; so (A ∩ B)′ = A′ ∪ B′.

Explain the commutative, associative and distributive laws of sets.

State the commutative laws.

A ∪ B = B ∪ A and A ∩ B = B ∩ A — the order of sets does not matter.

State the associative laws.

$(A \cup B) \cup C = A \cup (B \cup C)$ and $(A \cap B) \cap C = A \cap (B \cap C)$.

State the distributive laws.

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ and $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

Verify $A \cap (B \cup C)$ for $A = \{1, 2, 3\}$, $B = \{2, 4\}$, $C = \{3, 5\}$.

$B \cup C = \{2, 3, 4, 5\}$, so $A \cap (B \cup C) = \{2, 3\}$; $(A \cap B) \cup (A \cap C) = \{2\} \cup \{3\} = \{2, 3\}$.

Describe Venn diagrams and their use.

How is a Venn diagram drawn?

A rectangle for the universal set and circles inside it for the sets; the required region is shaded.

How are overlapping, disjoint and subset cases shown?

Overlapping circles share a region; disjoint circles do not touch; a subset circle lies wholly inside the other.

Which region shows A – B?

The part of circle A outside circle B.

In a class, 22 like tea, 15 like coffee and 7 like both. How many like only tea?

22 – 7 = 15 like only tea.

Multiple Choice Questions (MCQs)

$\{x \mid x \in N, x < 4\}$ in tabular form is: (A) {0, 1, 2, 3} (B) {1, 2, 3} (C) {1, 2, 3, 4} (D) {2, 3}
Correct answer: (B) {1, 2, 3}. N starts from 1.

{p, q, r} and {1, 2, 3} are: (A) Equal (B) Equivalent (C) Disjoint only (D) Empty
Correct answer: (B) Equivalent. Same number of elements.

The empty set is a subset of: (A) No set (B) Only itself (C) Every set (D) Only N
Correct answer: (C) Every set. Every set.

If A has 3 elements, P(A) has: (A) 3 (B) 6 (C) 8 (D) 9
Correct answer: (C) 8. 2^3.

P(∅) has how many elements? (A) 0 (B) 1 (C) 2 (D) Infinite
Correct answer: (B) 1. {∅}.

$A \cup B$ contains elements in: (A) Both only (B) A or B or both (C) A only (D) Neither
Correct answer: (B) A or B or both. Union.

If $A = \{1, 2, 3\}$, $B = \{3, 4\}$, then $A \cap B$ = (A) {3} (B) {1, 2} (C) {1, 2, 3, 4} (D) ∅

Correct answer: (A) {3}. Common element.

For the same sets, A – B = (A) {4} (B) {1, 2} (C) {3} (D) {1, 2, 4}
Correct answer: (B) {1, 2}. In A not B.

A′ means: (A) U ∩ A (B) U – A (C) A – U (D) A ∪ U
Correct answer: (B) U – A. Complement.

If A ∩ B = ∅, A and B are: (A) Equal (B) Overlapping (C) Disjoint (D) Universal
Correct answer: (C) Disjoint. No common element.

(A ∪ B)′ = (A) A′ ∪ B′ (B) A′ ∩ B′ (C) A ∩ B (D) A ∪ B′
Correct answer: (B) A′ ∩ B′. De Morgan.

A ∩ B = B ∩ A is the ___ law. (A) Associative (B) Commutative (C) Distributive (D) De Morgan
Correct answer: (B) Commutative. Commutative.

If A ⊆ B, then A ∪ B = (A) A (B) B (C) ∅ (D) U
Correct answer: (B) B. B already contains A.

Which is true for A = {a, b}? (A) a ∈ P(A) (B) {a} ∈ P(A) (C) {a} ∈ A (D) ∅ ∈ A
Correct answer: (B) {a} ∈ P(A). Elements of P(A) are subsets.

n(A) = 12, n(B) = 9, n(A ∩ B) = 4, so n(A ∪ B) = (A) 17 (B) 21 (C) 25 (D) 13
Correct answer: (A) 17. 12 + 9 – 4.

Quick Revision Summary

- Set = well-defined collection; forms: tabular, descriptive, set-builder.
- Equal = same elements; equivalent = same number of elements.
- A ⊆ B; ∅ is a subset of every set; P(A) has 2^n elements.
- ∪ = or, ∩ = both, A – B = only A, A′ = U – A; disjoint means A ∩ B = ∅.
- Commutative, associative and distributive laws hold for ∪ and ∩.
- De Morgan: (A ∪ B)′ = A′ ∩ B′; (A ∩ B)′ = A′ ∪ B′.
- n(A ∪ B) = n(A) + n(B) – n(A ∩ B).

Exam Tips

- Never repeat an element when listing a set.
- When listing a power set, start with ∅ and end with the set itself; count to check 2^n.
- For long verification questions, write each side step by step and compare at the end.
- Shade Venn diagrams lightly and label U, A and B clearly.
- In word problems, subtract the 'both' group once to avoid counting it twice.

Unit 5: Ratio, Rate and Proportion

Unit 5 studies how two or more quantities change together. A ratio compares two quantities of the same kind, a rate compares quantities of different kinds (such as rupees per kilogram or kilometres per hour), and a proportion states that two ratios are equal.

These notes explain direct and inverse proportion using tables, equations and graphs, and then compound proportion, where three or more quantities are linked, with many real-life word problems on cost, work, speed and time.

Learning Objectives

- Distinguish direct and inverse proportion with examples.
- Solve direct proportion problems using a table, an equation $y = kx$ and a graph.
- Solve inverse proportion problems using a table, an equation $xy = k$ and a graph.
- Solve compound proportion problems in real-life situations.

Key Concepts

5.1 Ratio, Rate and Proportion

A ratio $a : b$ compares two quantities of the same kind and has no unit. A rate compares quantities of different kinds, e.g., Rs. 250 per kg or 60 km per hour.

A proportion is an equality of two ratios, $a : b = c : d$. Here a and d are the extremes, b and c are the means, and the product of extremes equals the product of means ($ad = bc$).

5.2 Direct Proportion

Two quantities are in direct proportion if both increase or both decrease in the same ratio, such as the number of eggs and their cost.

Table method: apply the same operation to both quantities. If 5 kg of sugar costs Rs. 1250, then 1 kg costs Rs. 250 (÷ 5) and 8 kg cost Rs. 2000 (× 8).

Equation method: $y \propto x$ means $y = kx$, where k is the constant of proportionality. From $1250 = k \times 5$, $k = 250$, so for $x = 8$, $y = 2000$.

Graph: the graph of a direct proportion is a straight line through the origin; its steepness shows k.

5.3 Inverse Proportion

Two quantities are in inverse proportion if one increases when the other decreases in the same ratio, such as speed and time for a fixed distance, or workers and days for a fixed job.

Table method: if one quantity is multiplied by a number, the other is divided by the same number. If 12 workers build a wall in 10 days, then 24 workers take 5 days.

Equation method: $y \propto 1/x$ means $xy = k$. From $12 \times 10 = 120$, for 15 workers the days are $120 \div 15$

= 8.

Graph: the graph of an inverse proportion is a curve (hyperbola) that falls as x increases and never touches the axes.

5.4 Compound Proportion

In compound proportion one quantity depends on two or more other quantities. Compare the unknown quantity with each other quantity separately, decide whether each relation is direct or inverse, and combine.

Direct-direct example: if 6 men earn Rs. 36,000 in 5 days, each man earns Rs. 1200 per day, so 9 men earn 9 × 8 × 1200 = Rs. 86,400 in 8 days.

With an inverse part: 10 workers working 8 hours a day finish a job in 12 days (960 worker-hours). 16 workers working 6 hours a day give 96 worker-hours a day, so they need 960 ÷ 96 = 10 days.

Important Definitions

Ratio
A comparison of two quantities of the same kind, a : b.

Rate
A comparison of quantities of different kinds, e.g., km per hour.

Proportion
An equality of two ratios, a : b = c : d.

Extremes and means
In a : b = c : d, a and d are extremes; b and c are means.

Direct proportion
Both quantities increase or decrease in the same ratio; y = kx.

Inverse proportion
One quantity increases as the other decreases in the same ratio; xy = k.

Constant of proportionality
The fixed number k in y = kx or xy = k.

Compound proportion
A relation in which one quantity depends on two or more others.

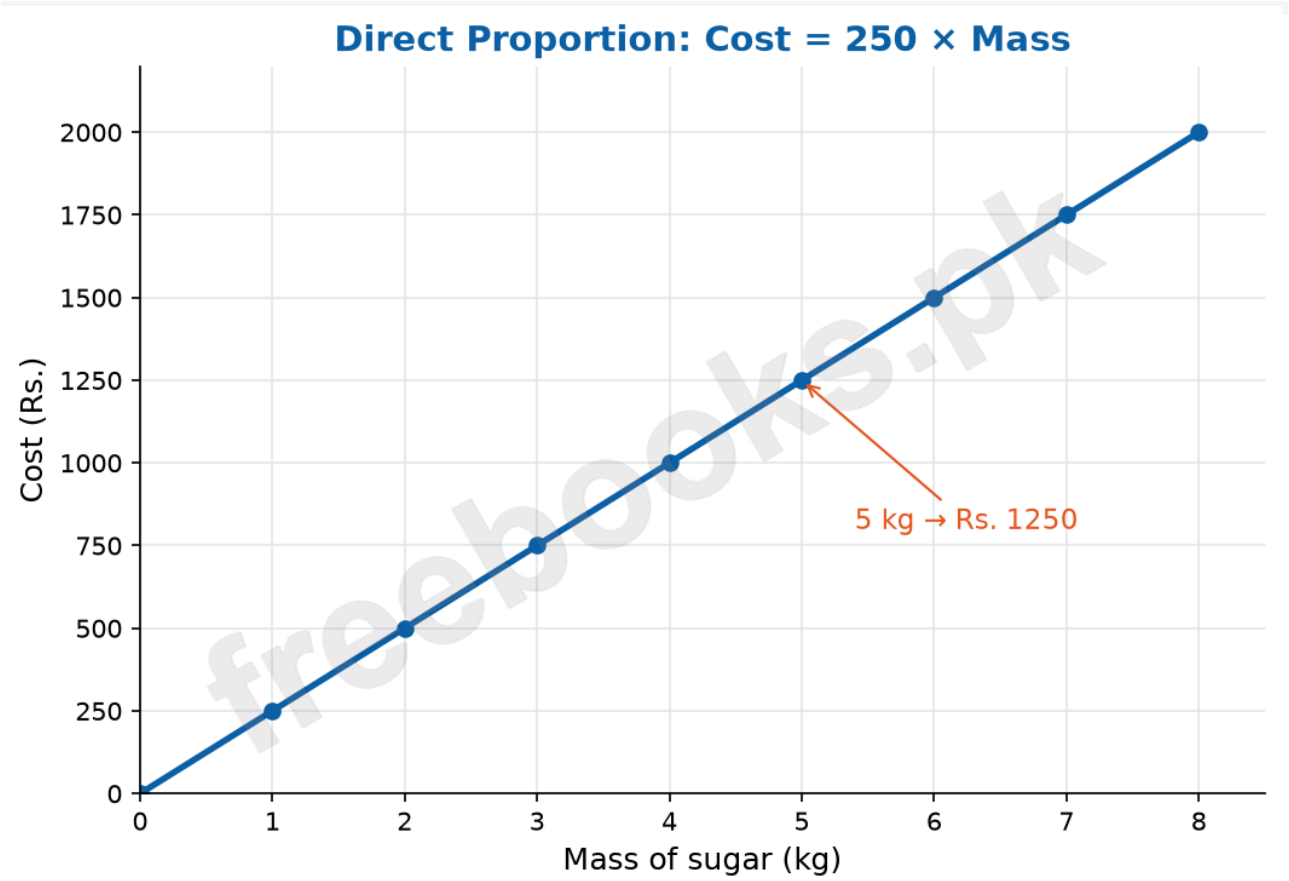
Unitary method
Finding the value of one unit first, then the required number of units.

Key Facts at a Glance

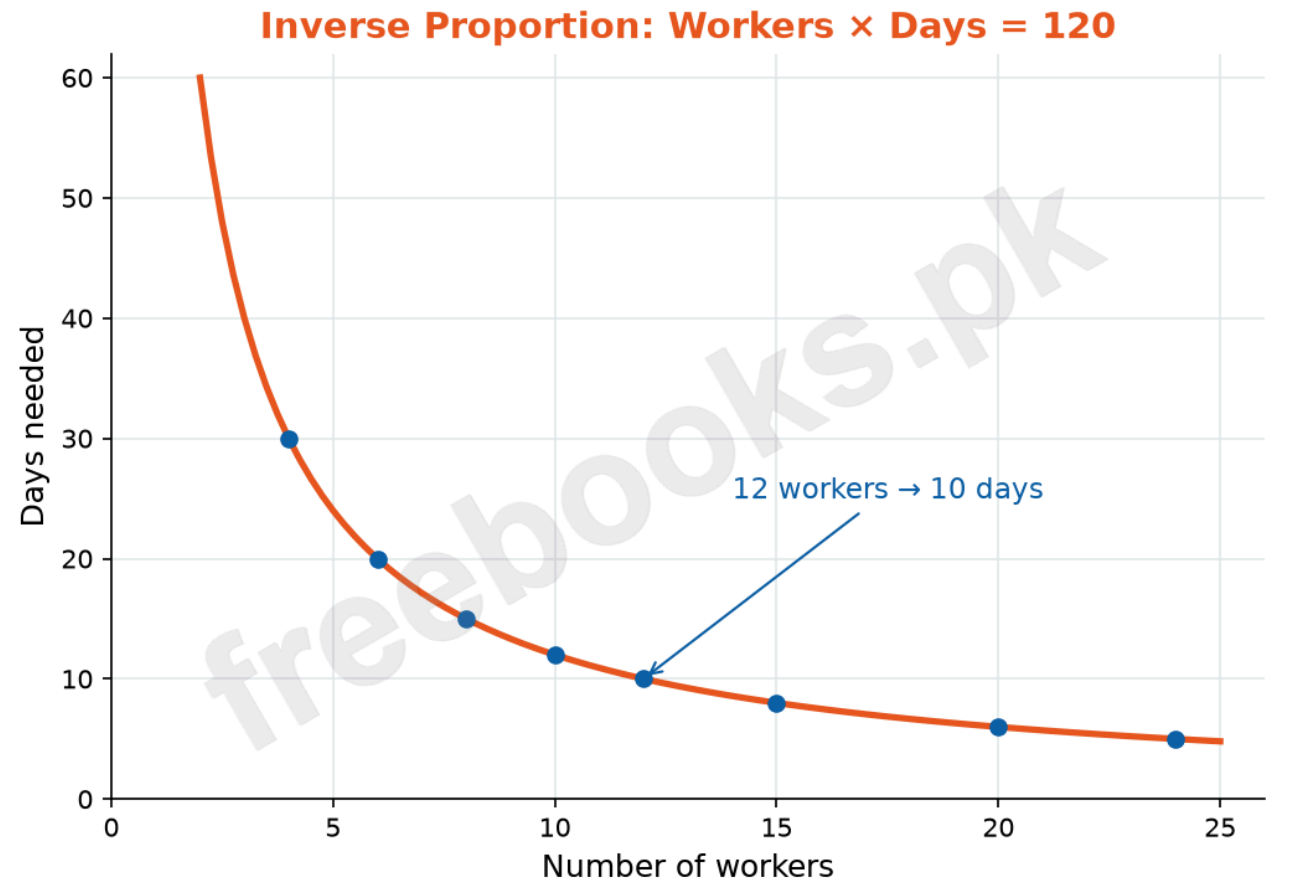
Relation	Rule / example
Direct proportion	$y = kx$ ($y \propto x$)
Inverse proportion	$xy = k$ ($y \propto 1/x$)
Proportion a : b = c : d	$ad = bc$
Direct graph	Straight line through origin
Inverse graph	Falling curve (hyperbola)
Cost and quantity	Direct
Speed and time (fixed distance)	Inverse
Workers and days (fixed work)	Inverse
Distance and time (fixed speed)	Direct
5 kg = Rs. 1250 → 8 kg	Rs. 2000
12 workers 10 days → 15 workers	8 days
10 workers × 8 h × 12 days	960 worker-hours

Diagrams

Graph of direct proportion: Cost of sugar against mass: a straight line through the origin ($y = 250x$).

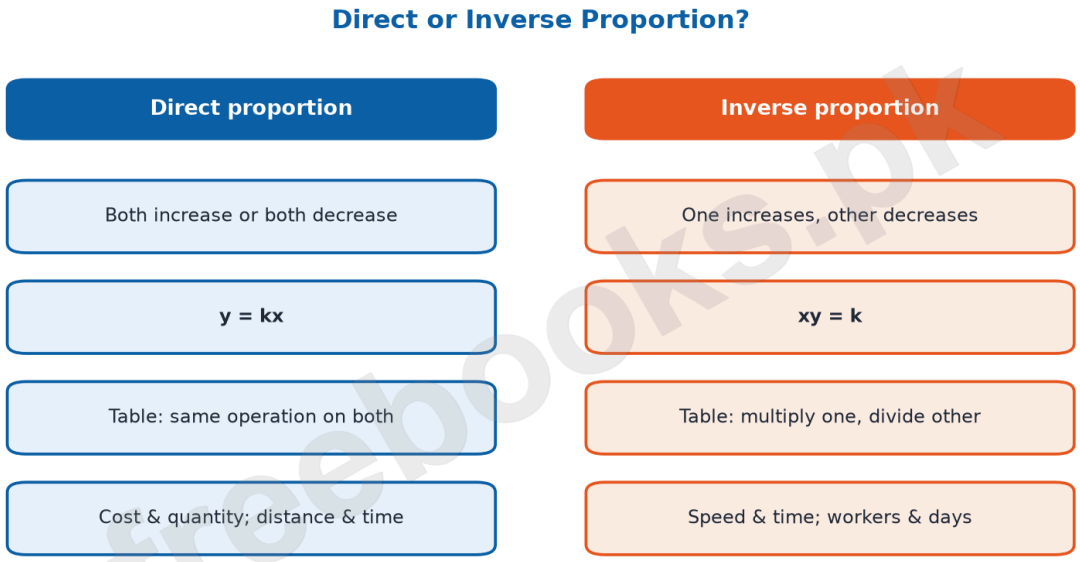


Graph of inverse proportion: Days needed against number of workers for a fixed job: a falling curve (xy = 120).



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Direct or inverse?: A chart comparing direct and inverse proportion with table rules and examples.



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Short Questions and Answers

Distinguish ratio and rate.
A ratio compares quantities of the same kind with no unit; a rate compares different kinds, such as Rs. per kg.

Find x if 3 : 5 = x : 20.
5x = 60, so x = 12.

Are the number of pens and their cost directly or inversely proportional?
Directly proportional.

If 4 pens cost Rs. 180, find the cost of 10 pens.
1 pen = Rs. 45, so 10 pens = Rs. 450.

y varies directly as x and y = 21 when x = 3. Find y when x = 7.
k = 7, so y = 49.

What does the graph of a direct proportion look like?
A straight line through the origin.

Are speed and time inversely proportional for a fixed distance? Why?
Yes; doubling the speed halves the time because speed × time = distance stays fixed.

8 taps fill a tank in 6 hours. How long will 12 taps take?

8 × 6 = 48, so 48 ÷ 12 = 4 hours.

y varies inversely as x and y = 9 when x = 4. Find y when x = 12.
k = 36, so y = 3.

A car at 60 km/h takes 5 hours. How long at 75 km/h?
60 × 5 = 300 km, so 300 ÷ 75 = 4 hours.

What does the graph of an inverse proportion look like?
A curve that falls as x increases and does not meet the axes.

What is compound proportion?
A relation in which one quantity depends on two or more other quantities.

A hostel's food for 120 students lasts 30 days. How long for 150 students?
120 × 30 = 3600 student-days, so 3600 ÷ 150 = 24 days.

If 5 machines make 400 bottles in 2 hours, how many bottles will 8 machines make in 3 hours?
Per machine-hour 40 bottles, so 8 × 3 × 40 = 960 bottles.

State the rule for an inverse proportion table.
If one quantity is multiplied by a number, the other is divided by the same number.

Long Questions and Answers

Explain direct proportion using a table, an equation and a graph.

What is direct proportion?

A relation in which both quantities rise or fall in the same ratio, e.g., hours worked and wages.

How is a table used?

The same operation is applied to both columns: if 3 hours earn Rs. 900, 1 hour earns Rs. 300 and 7 hours earn Rs. 2100.

How is the equation used?

Write y = kx, find k from known values (900 = 3k, k = 300), then substitute (y = 300 × 7 = 2100).

What does the graph show?

A straight line through the origin; every point satisfies y = 300x.

Explain inverse proportion using a table, an equation and a graph.

What is inverse proportion?

When one quantity increases the other decreases in the same ratio, e.g., workers and days for a

fixed job.

How is a table used?

Multiply one quantity and divide the other by the same number: 6 workers 20 days → 12 workers 10 days.

How is the equation used?

xy = k: 6 × 20 = 120, so 8 workers need 120 ÷ 8 = 15 days.

What does the graph show?

A falling curve; the product of the coordinates of every point is 120.

Solve compound proportion problems.

What is the method?

Compare the unknown quantity with each other quantity, decide direct or inverse, and combine using a constant product or rate.

15 labourers dig a 60 m trench in 4 days. How many days will 20 labourers take to dig 100 m?

Rate = 60 ÷ (15 × 4) = 1 m per labourer-day; 100 ÷ 20 = 5 days.

A family of 6 uses 90 kg of flour in 30 days. How much will a family of 8 use in 45 days?

Per person-day 0.5 kg, so 8 × 45 × 0.5 = 180 kg.

12 workers working 6 hours a day finish in 15 days. How many days for 9 workers at 8 hours a day?

Total 1080 worker-hours; per day 72, so 15 days.

Identify whether each situation is direct or inverse proportion and solve it.

Distance and petrol: 15 litres for 180 km. Petrol for 300 km?

Direct: 1 litre per 12 km, so 25 litres.

Speed and time: 4 hours at 90 km/h. Time at 72 km/h?

Inverse: 360 ÷ 72 = 5 hours.

Number of pipes and time: 5 pipes take 9 hours. Time for 3 pipes?

Inverse: 45 ÷ 3 = 15 hours.

Rice and cost: 25 kg cost Rs. 7500. Cost of 40 kg?

Direct: Rs. 300 per kg, so Rs. 12,000.

Explain ratio, rate and the property of a proportion.

Write the ratio 45 minutes : 2 hours in simplest form.

45 min : 120 min = 3 : 8.

What is a unit rate? Give an example.

The rate for one unit: Rs. 840 for 12 eggs gives Rs. 70 per egg.

State the product rule of a proportion.

In a : b = c : d, the product of the extremes equals the product of the means: ad = bc.

Check whether 4 : 6 and 10 : 15 are in proportion.

4 × 15 = 60 and 6 × 10 = 60, so they are in proportion.

Multiple Choice Questions (MCQs)

A ratio compares quantities of: (A) Different kinds (B) The same kind (C) Only money (D) Only time

Correct answer: (B) The same kind. Same kind.

60 km per hour is a: (A) Ratio (B) Rate (C) Proportion (D) Percentage

Correct answer: (B) Rate. Different units.

If 2 : 7 = 6 : x, then x = (A) 14 (B) 21 (C) 18 (D) 24

Correct answer: (B) 21. 2x = 42.

Cost and quantity of an item are in: (A) Inverse proportion (B) Direct proportion (C) No relation (D) Compound

Correct answer: (B) Direct proportion. Direct.

Direct proportion can be written as: (A) xy = k (B) y = kx (C) y = k/x (D) x + y = k

Correct answer: (B) y = kx. y = kx.

Inverse proportion can be written as: (A) y = kx (B) xy = k (C) y = x + k (D) y = k – x

Correct answer: (B) xy = k. xy = k.

The graph of a direct proportion is: (A) A curve (B) A straight line through origin (C) A circle (D) A horizontal line

Correct answer: (B) A straight line through origin. Line through origin.

If 3 books cost Rs. 450, 7 books cost: (A) Rs. 900 (B) Rs. 1050 (C) Rs. 1150 (D) Rs. 1350

Correct answer: (B) Rs. 1050. 150 × 7.

If 4 workers finish a job in 9 days, 6 workers take: (A) 4 days (B) 6 days (C) 8 days (D) 13.5 days

Correct answer: (B) 6 days. 36 ÷ 6.

For a fixed distance, speed and time are: (A) Directly proportional (B) Inversely proportional (C) Equal (D) Unrelated

Correct answer: (B) Inversely proportional. Inverse.

If $y \propto x$ and $y = 10$ when $x = 2$, then $k =$ (A) 2 (B) 5 (C) 10 (D) 20
Correct answer: (B) 5. $10 \div 2$.

If $xy = 48$ and $x = 6$, then $y =$ (A) 6 (B) 8 (C) 42 (D) 288
Correct answer: (B) 8. $48 \div 6$.

In inverse proportion, if x is doubled, y is: (A) Doubled (B) Halved (C) Unchanged (D) Squared
Correct answer: (B) Halved. Halved.

A relation linking three quantities is: (A) Simple proportion (B) Compound proportion (C) Ratio (D) Rate
Correct answer: (B) Compound proportion. Compound.

2 machines make 100 items in 5 hours. 4 machines in 5 hours make: (A) 50 (B) 100 (C) 200 (D) 400
Correct answer: (C) 200. Direct with machines.

Quick Revision Summary

- Ratio: same kind (no unit); rate: different kinds; proportion: $a : b = c : d$, $ad = bc$.
- Direct: $y = kx$; same operation on both; straight-line graph through origin.
- Inverse: $xy = k$; multiply one, divide the other; falling curve.
- Examples — direct: cost/quantity, distance/time at fixed speed; inverse: speed/time, workers/days.
- Compound: compare with each quantity separately; use worker-hours or per-unit rates.

Exam Tips

- Before solving, write 'direct' or 'inverse' next to each pair of quantities.
- Use the unitary method: find the value for one unit first.
- In compound problems, find a combined unit such as worker-days or machine-hours.
- Check your answer for sense: more workers should mean fewer days.
- Draw graphs on squared paper with labelled axes and a clear scale.

Unit 6: Percentage and Financial Arithmetic

Unit 6 applies percentages to everyday money matters. Percent means 'per hundred', so 15% = 15/100. Shops, banks, insurance companies and families all use percentages to work out profit, loss, discounts, bank profit or markup, premiums and shares.

These notes cover currency conversion, profit and loss percentages, discount, successive transactions, bank profit and markup (finding profit, principal, rate or period), life and vehicle insurance, partnership, and distribution of inheritance according to Islamic principles.

Learning Objectives

- Convert Pakistani rupees to other currencies and back using a given rate.
- Calculate profit percentage, loss percentage and percentage discount.
- Solve problems with successive transactions (marked price, discount, profit or loss).
- Distinguish bank profit and markup and find profit/markup, principal, rate or period.
- Solve simple problems on life and vehicle insurance.
- Divide profit among partners and distribute inheritance using Islamic principles.

Key Concepts

6.1 Currency Conversion

Different countries use different currencies, and the exchange rate tells how many rupees equal one unit of another currency. Exchange rates change daily, so problems always state the rate to use.

To change foreign currency to rupees, multiply by the rate; to change rupees to foreign currency, divide by the rate. Example (taking 1 US dollar = Rs. 280): 50 dollars = 50 × 280 = Rs. 14,000, and Rs. 56,000 = 56,000 ÷ 280 = 200 dollars.

6.2 Profit, Loss and Discount

Cost price (CP) is what the seller pays; selling price (SP) is what the buyer pays. Profit = SP − CP (when SP > CP); loss = CP − SP (when CP > SP).

Profit % = (profit ÷ CP) × 100 and loss % = (loss ÷ CP) × 100 — both are always worked out on the cost price.

The marked price (MP) is the price written on the item. Discount = MP − SP and discount % = (discount ÷ MP) × 100.

Successive transaction example: CP = Rs. 6000 and MP is 15% above CP, so MP = Rs. 6900. If the item is sold for Rs. 4600, the discount is Rs. 2300, which is 2300/6900 × 100 ≈ 33.33%; the seller's loss is Rs. 1400, i.e., 1400/6000 × 100 ≈ 23.33%.

6.3 Bank Profit and Markup

When we deposit money, the bank pays us an extra amount called profit. When we borrow money, the bank charges an extra amount called markup.

The amount deposited or borrowed is the principal (P), the yearly percentage is the profit/markup rate (R), and the time in years is the period (T).

Profit or markup = P × R × T ÷ 100. Rearranging: P = (profit × 100) ÷ (R × T), R = (profit × 100) ÷ (P × T), T = (profit × 100) ÷ (P × R).

Example: Rs. 50,000 at 8% for 3 years gives 50,000 × 8 × 3 ÷ 100 = Rs. 12,000.

6.4 Insurance

Insurance is a contract that protects against financial loss from risks such as death, illness, accident or damage. The written contract is the policy; the company bearing the risk is the insurer and the person covered is the insured. The insured pays regular instalments called premiums.

Life insurance: the company pays the agreed amount (with profit) at the end of the period, or earlier on death or critical illness. Example: a Rs. 8,00,000 policy with a 5% annual premium and a 0.5% policy fee costs 40,000 + 4,000 = Rs. 44,000 a year.

Vehicle insurance: the premium is a percentage of the vehicle's value. A car worth Rs. 20,00,000 insured at 1.5% has a premium of Rs. 30,000.

6.5 Partnership

In a partnership, two or more people invest capital in a business and share profit or loss in the ratio of their investments.

If capitals are invested for the same time, share in the ratio of capitals: A Rs. 30,000 and B Rs. 45,000 → 2 : 3, so a profit of Rs. 15,000 gives A Rs. 6,000 and B Rs. 9,000.

If times differ, multiply each capital by its time and use those products as the ratio.

6.6 Inheritance (Islamic Principles)

When a person dies, the property left (the estate) is distributed among the legal heirs according to Islamic law. First the funeral expenses and the debts owed by the deceased are paid; then any will is carried out up to one-third of the remaining property; then the rest is divided among the heirs.

When the deceased man leaves children, the wife's share is 1/8. The remaining property is shared by sons and daughters, with each son receiving twice the share of each daughter (ratio 2 : 1).

Example: estate Rs. 9,00,000; debts Rs. 60,000 and funeral Rs. 15,000 leave Rs. 8,25,000. Wife gets 1/8 = Rs. 1,03,125; the remaining Rs. 7,21,875 is shared by 1 son and 1 daughter in the ratio 2 : 1, so the son gets Rs. 4,81,250 and the daughter Rs. 2,40,625.

Important Definitions

Exchange rate
The value of one unit of a foreign currency in rupees.

Cost price (CP)
The price at which an item is bought.

Selling price (SP)
The price at which an item is sold.

Marked price (MP)
The price written on an item before discount.

Profit percentage
(Profit ÷ CP) × 100.

Loss percentage
(Loss ÷ CP) × 100.

Discount
The reduction from the marked price; MP – SP.

Bank profit
The extra amount a bank pays on a deposit.

Markup
The extra amount a bank charges on a loan.

Principal
The amount deposited or borrowed.

Insurance policy
The written contract between insurer and insured.

Premium
The regular payment made for an insurance policy.

Partnership
A business owned jointly by two or more people who share profit or loss.

Inheritance
Property left by a deceased person, distributed among heirs.

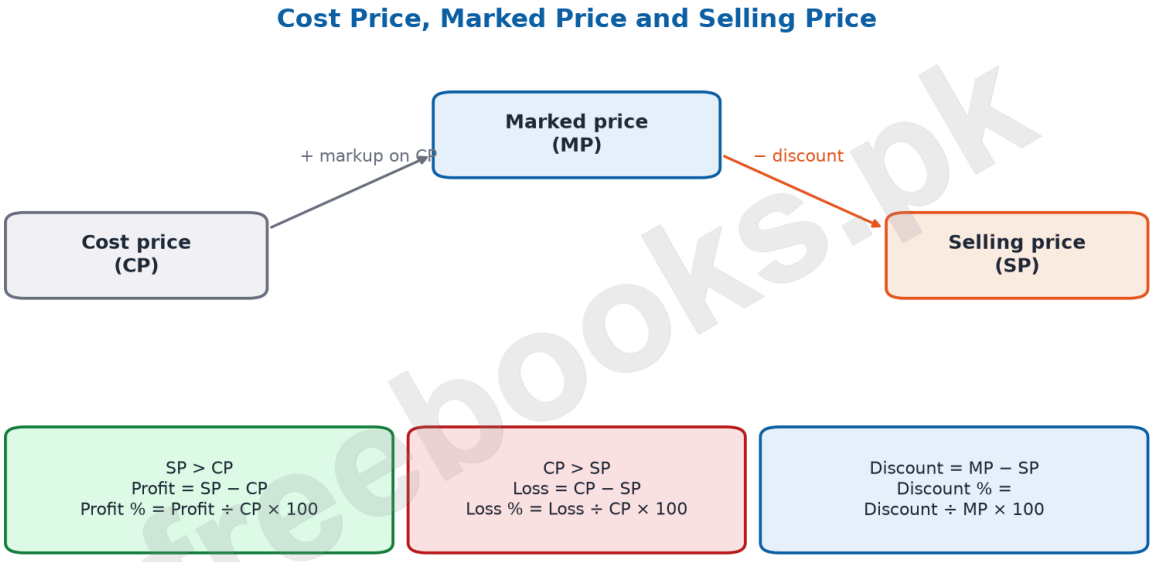
Key Facts at a Glance

Quantity	Formula
Profit	SP – CP

Loss	CP – SP
Profit %	Profit ÷ CP × 100
Discount %	Discount ÷ MP × 100
SP after r% discount	MP × (100 – r) ÷ 100
Profit / markup	P × R × T ÷ 100
Principal	Profit × 100 ÷ (R × T)
Rate	Profit × 100 ÷ (P × T)
Period	Profit × 100 ÷ (P × R)
Annual premium	Premium % of policy + policy fee
Partnership share	Ratio of capital (× time)
Wife's share (with children)	1/8; son : daughter = 2 : 1

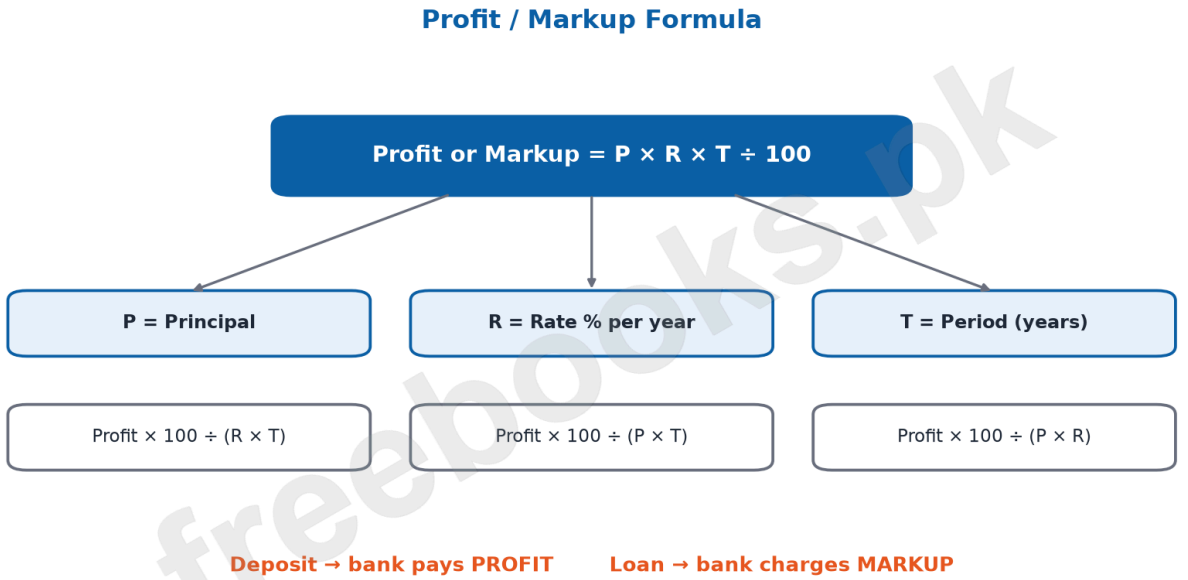
Diagrams

Profit, loss and discount: A chart linking cost price, marked price and selling price with the formulas for profit, loss and discount.

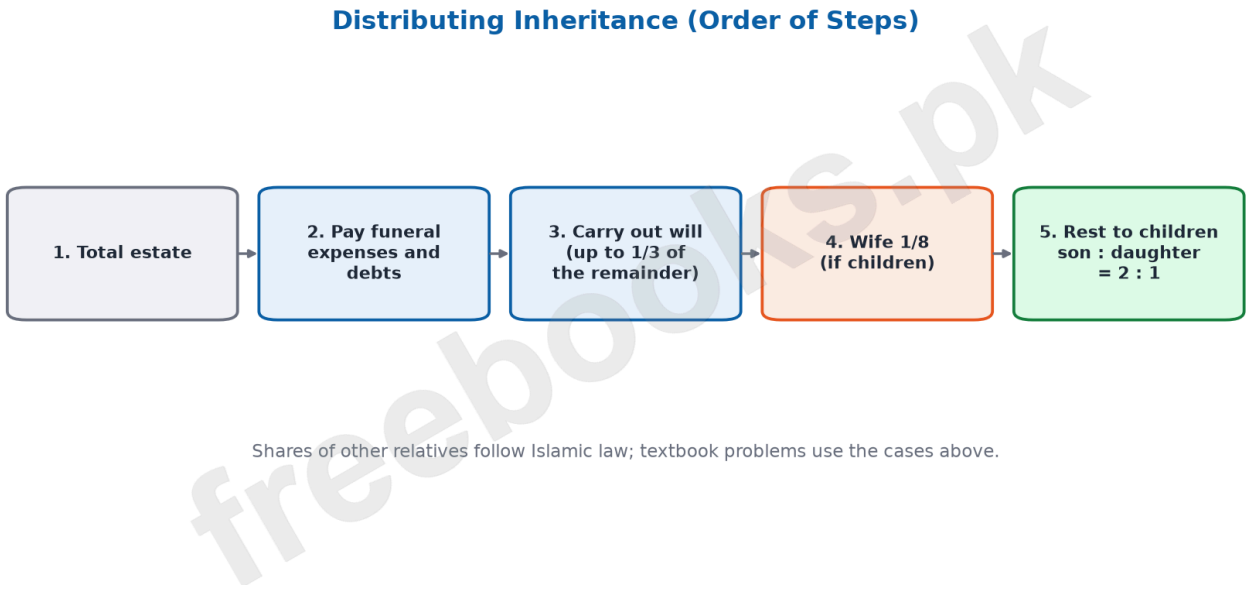


Profit or markup formula: The formula triangle for profit/markup with principal, rate and period.

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Order of distributing inheritance: A flow chart: funeral expenses and debts, will up to one-third, then shares of heirs.



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Short Questions and Answers

Convert 75 Saudi riyals to rupees if 1 riyal = Rs. 74.
75 × 74 = Rs. 5,550.

Convert Rs. 42,000 to dollars if 1 dollar = Rs. 280.
42,000 ÷ 280 = 150 dollars.

A bag bought for Rs. 1200 is sold for Rs. 1500. Find profit %.
Profit Rs. 300; 300/1200 × 100 = 25%.

A phone bought for Rs. 25,000 is sold for Rs. 22,000. Find loss %.
Loss Rs. 3000; 3000/25,000 × 100 = 12%.

On which price are profit and loss percentages calculated?
On the cost price.

A shirt marked Rs. 2000 is sold for Rs. 1700. Find the discount %.
Discount Rs. 300; 300/2000 × 100 = 15%.

Find the sale price of a Rs. 3,600 item after a 25% discount.
3600 × 75/100 = Rs. 2,700.

Distinguish bank profit and markup.
Profit is paid by the bank on deposits; markup is charged by the bank on loans.

Find the profit on Rs. 40,000 at 7% for 2 years.
40,000 × 7 × 2 ÷ 100 = Rs. 5,600.

A sum earns Rs. 9,000 profit at 6% in 3 years. Find the principal.
9,000 × 100 ÷ (6 × 3) = Rs. 50,000.

Rs. 25,000 earns Rs. 7,500 at 10%. Find the period.
7,500 × 100 ÷ (25,000 × 10) = 3 years.

What is a premium?
The regular payment made to the insurance company to keep a policy in force.

A motorcycle worth Rs. 2,40,000 is insured at 2%. Find the premium.
2,40,000 × 2/100 = Rs. 4,800.

A and B invest Rs. 20,000 and Rs. 30,000 for the same time. Divide a profit of Rs. 10,000.
Ratio 2 : 3; A Rs. 4,000, B Rs. 6,000.

What is paid first from the estate of a deceased person?
Funeral expenses and the debts owed by the deceased.

Long Questions and Answers

Explain profit, loss and discount with formulas and examples.

How are profit and loss found?

Profit = SP − CP when SP is more; loss = CP − SP when CP is more.

How are percentages found?

Profit % or loss % = (profit or loss ÷ CP) × 100.

What is discount?

The reduction on the marked price: discount = MP − SP, discount % = discount ÷ MP × 100.

A fan costs a shop Rs. 8,000, is marked 25% above cost and sold at 10% discount. Find the profit %.

MP Rs. 10,000; SP Rs. 9,000; profit Rs. 1,000 = 12.5%.

Explain bank profit and markup and how to find each quantity.

What do the terms mean?

Principal is the amount deposited or borrowed; rate is the yearly percentage; period is the time in years.

Write the formula.

Profit or markup = P × R × T ÷ 100.

Rs. 1,20,000 is borrowed at 12% markup for 2 years. Find the markup and total repayment.

Markup Rs. 28,800; total Rs. 1,48,800.

Rs. 40,000 earns Rs. 6,000 in 2 years. Find the rate.

6,000 × 100 ÷ (40,000 × 2) = 7.5%.

Explain insurance and solve problems on life and vehicle insurance.

What is insurance?

A contract in which the insurer protects the insured against financial loss from specified risks in return for premiums.

What is life insurance?

A policy that pays an agreed sum with profit at the end of the period or on the death or critical illness of the insured.

Find the annual premium of a Rs. 6,00,000 life policy at 4% with a 0.25% policy fee.

24,000 + 1,500 = Rs. 25,500.

A car worth Rs. 30,00,000 is insured at 1.25%. Find the premium.

Rs. 37,500.

Explain partnership with examples.

What is a partnership?

A business run jointly by partners who share profit or loss in the ratio of their investments.

How is profit shared when times are equal?

In the ratio of the capitals invested.

How is profit shared when times differ?

In the ratio of capital × time for each partner.

A invests Rs. 40,000 for 6 months and B Rs. 30,000 for 12 months. Divide Rs. 18,000 profit.

Products 2,40,000 : 3,60,000 = 2 : 3; A Rs. 7,200, B Rs. 10,800.

Explain the distribution of inheritance according to Islamic principles.

What is paid before the heirs' shares?

Funeral expenses and debts owed by the deceased, then any will up to one-third of the remaining property.

What is the wife's share when there are children?

One-eighth of the remaining estate.

How do sons and daughters share?

Each son gets twice the share of each daughter.

A man leaves Rs. 12,00,000 with Rs. 80,000 debt and Rs. 20,000 funeral expenses; heirs are his wife, 2 sons and 1 daughter. Find the shares.

Remaining Rs. 11,00,000; wife Rs. 1,37,500; rest Rs. 9,62,500 in parts 2 + 2 + 1 = 5; each son Rs. 3,85,000, daughter Rs. 1,92,500.

Solve problems on currency and successive transactions.

A traveller changes Rs. 1,40,000 to dollars at Rs. 280 each. How many dollars?

500 dollars.

He spends 320 dollars and changes the rest back at Rs. 278. How many rupees?

180 × 278 = Rs. 50,040.

A shopkeeper buys at Rs. 5,000, marks 20% above cost and gives 5% discount. Find SP.

MP Rs. 6,000; SP Rs. 5,700.

Find his profit %.

Profit Rs. 700 = 14%.

Multiple Choice Questions (MCQs)

Percent means: (A) Per ten (B) Per hundred (C) Per thousand (D) Per unit
Correct answer: (B) Per hundred. Per hundred.

If 1 dollar = Rs. 280, then 20 dollars = (A) Rs. 560 (B) Rs. 5,600 (C) Rs. 2,800 (D) Rs. 14

Correct answer: (B) Rs. 5,600. 20 × 280.

Profit % is calculated on: (A) SP (B) MP (C) CP (D) Discount
Correct answer: (C) CP. Cost price.

CP Rs. 800, SP Rs. 1000. Profit % = (A) 20% (B) 25% (C) 15% (D) 30%
Correct answer: (B) 25%. 200/800.

CP Rs. 500, SP Rs. 450. Loss % = (A) 5% (B) 10% (C) 11% (D) 50%
Correct answer: (B) 10%. 50/500.

Discount is calculated on the: (A) Cost price (B) Marked price (C) Profit (D) Loss
Correct answer: (B) Marked price. Marked price.

MP Rs. 1500, discount 20%. SP = (A) Rs. 1200 (B) Rs. 1300 (C) Rs. 1250 (D) Rs. 300
Correct answer: (A) Rs. 1200. 80% of 1500.

Extra money charged by a bank on a loan is: (A) Profit (B) Markup (C) Premium (D) Discount
Correct answer: (B) Markup. Markup.

Profit on Rs. 10,000 at 5% for 2 years is: (A) Rs. 500 (B) Rs. 1,000 (C) Rs. 1,500 (D) Rs. 2,000
Correct answer: (B) Rs. 1,000. 10,000 × 5 × 2 ÷ 100.

Principal = Profit × 100 ÷ (A) (P × T) (B) (R × T) (C) (P × R) (D) R
Correct answer: (B) (R × T). Rearranged formula.

Regular payment for insurance is called: (A) Policy (B) Premium (C) Markup (D) Capital
Correct answer: (B) Premium. Premium.

The company that bears the risk is the: (A) Insured (B) Insurer (C) Partner (D) Heir
Correct answer: (B) Insurer. Insurer.

A and B invest in the ratio 3 : 2. Profit Rs. 5,000. A gets: (A) Rs. 2,000 (B) Rs. 3,000 (C) Rs. 2,500 (D) Rs. 3,500
Correct answer: (B) Rs. 3,000. 3/5 × 5000.

With children present, the wife's share is: (A) 1/4 (B) 1/8 (C) 1/3 (D) 1/2
Correct answer: (B) 1/8. One-eighth.

A will can be executed up to: (A) 1/2 (B) 1/3 (C) 1/4 (D) Whole
Correct answer: (B) 1/3. One-third.

Quick Revision Summary

- Foreign → rupees: multiply by rate; rupees → foreign: divide.
- Profit/loss % on CP; discount % on MP; SP = MP − discount.
- Profit/markup = PRT/100; rearrange for P, R or T.
- Insurance: policy, insurer, insured, premium; life and vehicle insurance.
- Partnership: share in ratio of capital (× time).
- Inheritance: funeral & debts → will (≤ 1/3) → heirs; wife 1/8 with children; son : daughter = 2 : 1.

Exam Tips

- Always write CP, MP and SP clearly before starting.
- Remember: profit/loss on CP, discount on MP.
- In PRT/100 questions, convert months to years (6 months = 1/2 year).
- For partnership with different times, multiply capital by months first.
- In inheritance questions, subtract funeral expenses and debts before any share.

Note on Facts

One textbook inheritance example says that a debt 'was due to him' and then subtracts it from the estate. Only debts owed by the deceased are paid out of the estate; money that others owed to the deceased would be added to it. Read such examples as debts owed by the deceased.

Unit 7: Number Sequence and Patterns

Unit 7 opens the Algebra domain with number sequences and patterns. A sequence is a list of numbers that follows a rule, such as 3, 7, 11, 15, ... Spotting the rule lets us continue the list, find any term directly, and model real situations such as savings plans, seating rows and growth.

These notes cover number sequences, arithmetic sequences (constant difference), the term-to-term and position-to-term rules, the general (nth) term formula, geometric sequences (constant ratio) and real-life problems involving patterns.

Learning Objectives

- Recognise a number sequence and describe its rule.
- Distinguish arithmetic and geometric sequences.
- Use the term-to-term rule to continue a sequence.
- Use the position-to-term rule and the nth-term formula.
- Find terms of a geometric sequence using $a_n = a \cdot r^{n-1}$.
- Solve real-life problems involving sequences and patterns.

Key Concepts

7.1 Number Sequences

A sequence is an ordered list of numbers called terms, written a_1, a_2, a_3, ... Each term has a position (1st, 2nd, 3rd, ...).

Sequences can come from number patterns (2, 4, 6, 8, ...), square numbers (1, 4, 9, 16, ...) or shape patterns (matchstick or dot figures that grow by a fixed amount).

7.2 Arithmetic Sequences

In an arithmetic sequence the difference between consecutive terms is constant. This common difference is $d = a_2 - a_1$. Example: 3, 7, 11, 15, ... has $d = 4$; 20, 17, 14, ... has $d = -3$.

Term-to-term rule: tells how to get the next term from the previous one, e.g., 'start at 3 and add 4'.

Position-to-term rule: tells how to get a term directly from its position n. For 3, 7, 11, ..., the nth term is $a_n = a + (n - 1)d = 3 + (n - 1)4 = 4n - 1$. So the 10th term is $4 \times 10 - 1 = 39$.

Going backwards: if the nth term is $5n - 2$, the sequence is 3, 8, 13, 18, ... (put n = 1, 2, 3, ...).

7.3 Geometric Sequences

In a geometric sequence each term is found by multiplying the previous term by a constant called the common ratio, $r = a_2 \div a_1$. Example: 2, 6, 18, 54, ... has $r = 3$; 80, 40, 20, ... has $r = 1/2$.

The nth term is $a_n = a \cdot r^{n-1}$. For 2, 6, 18, ..., $a_6 = 2 \times 3^5 = 486$.

Arithmetic sequences grow by adding the same amount; geometric sequences grow by multiplying

by the same amount, so they change much faster.

7.4 Real-Life Patterns

Seating: a hall has 20 seats in the first row and 2 more in each row. Row n has 20 + (n – 1)2 seats, so row 15 has 48 seats.

Savings: saving Rs. 500 in the first month and Rs. 100 more each month gives Rs. 1,600 in the 12th month.

Growth: bacteria that double every hour from 100 become $100 \times 2^5 = 3{,}200$ after 5 hours — a geometric pattern.

Important Definitions

Sequence
An ordered list of numbers that follows a rule.

Term
Each number in a sequence.

Arithmetic sequence
A sequence with a constant difference between consecutive terms.

Common difference (d)
The fixed amount added to get each next term: $d = a_2 - a_1$.

Geometric sequence
A sequence in which each term is multiplied by a constant to get the next.

Common ratio (r)
The fixed multiplier in a geometric sequence: $r = a_2 \div a_1$.

Term-to-term rule
A rule that gives the next term from the previous term.

Position-to-term rule
A rule that gives a term directly from its position n.

General term (nth term)
A formula for any term, e.g., $a_n = a + (n - 1)d$.

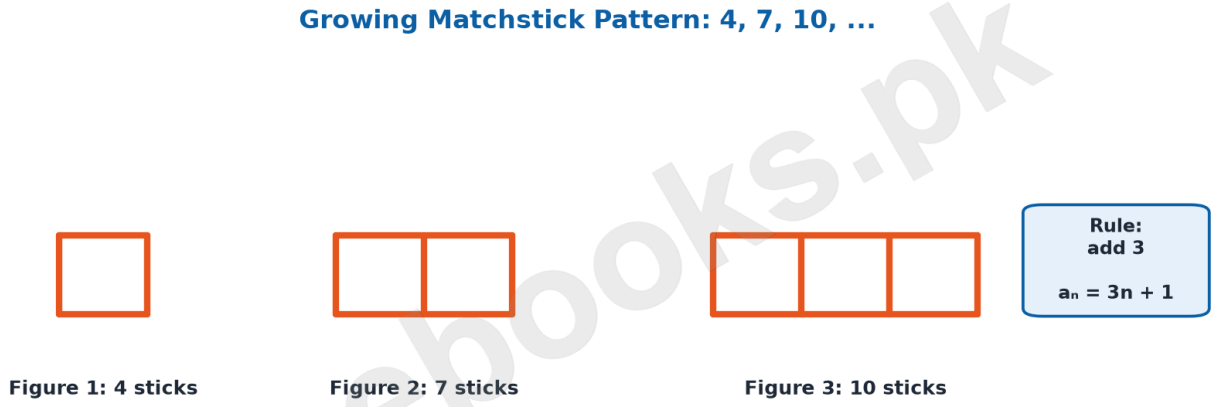
Key Facts at a Glance

Item	Formula / example

Common difference	$d = a_2 - a_1$
nth term (arithmetic)	$a_n = a + (n - 1)d$
Common ratio	$r = a_2 \div a_1$
nth term (geometric)	$a_n = a \cdot r^{n-1}$
3, 7, 11, ...	$a_n = 4n - 1$; $a_{10} = 39$
5n – 2	3, 8, 13, 18, ...
2, 6, 18, ...	$r = 3$; $a_6 = 486$
80, 40, 20, ...	$r = 1/2$
Square numbers	1, 4, 9, 16, 25, ...
Odd numbers	$a_n = 2n - 1$
Even numbers	$a_n = 2n$
Doubling from 100 (5 steps)	3,200

Diagrams

Growing matchstick pattern: A pattern of squares made of matchsticks that grows by 3 sticks each time (4, 7, 10, ...).



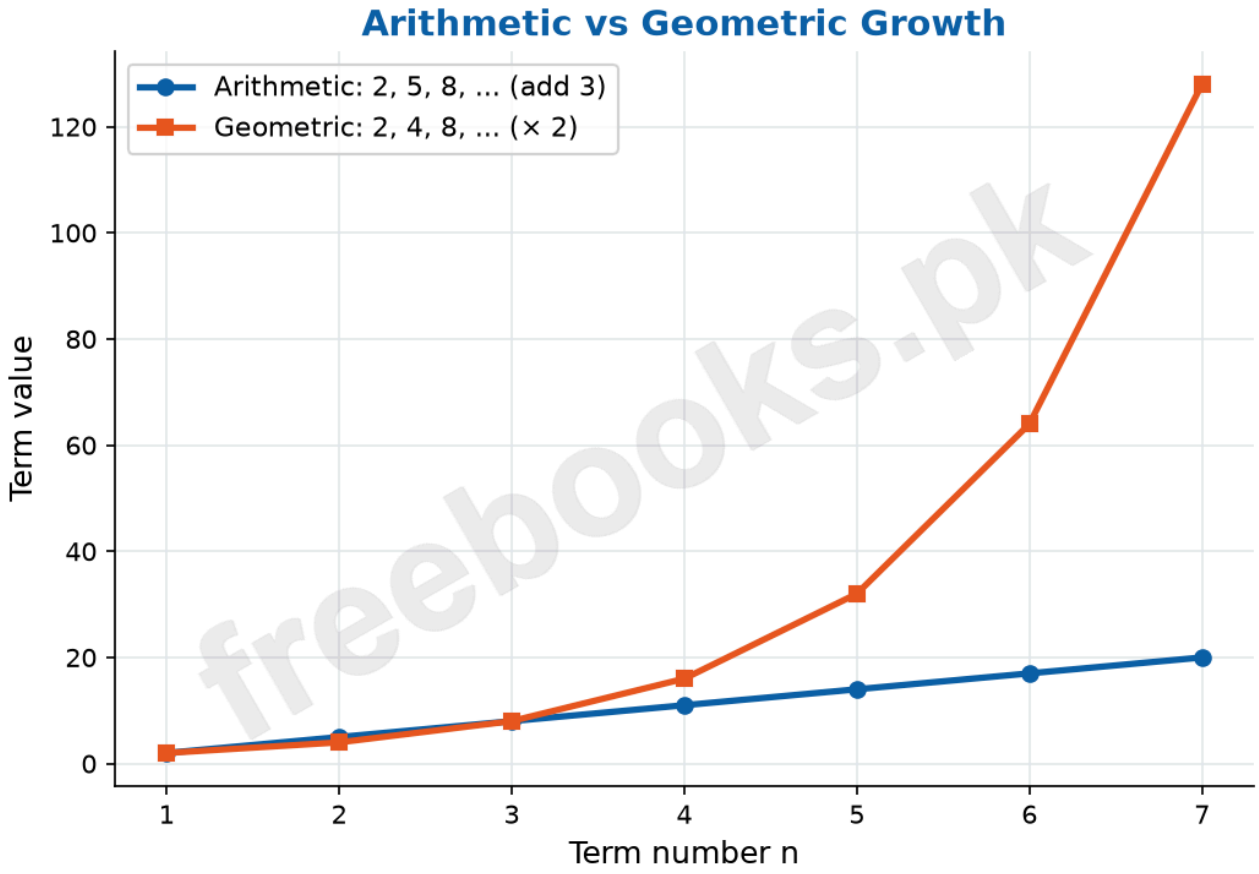
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Arithmetic vs geometric sequences: A comparison chart of arithmetic and geometric sequences with rules and examples.

Arithmetic vs Geometric Sequences

Arithmetic	Geometric
ADD the same number	MULTIPLY by the same number
Common difference $d = a_2 - a_1$	Common ratio $r = a_2 \div a_1$
$a_n = a + (n-1)d$	$a_n = a \cdot r^{n-1}$
3, 7, 11, 15, ... (d = 4)	2, 6, 18, 54, ... (r = 3)

Growth of sequences: A graph comparing the arithmetic sequence 2, 5, 8, ... with the geometric sequence 2, 4, 8, ...



Short Questions and Answers

What is a sequence?
An ordered list of numbers that follows a rule.

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Find the next two terms: 5, 11, 17, 23, ...
29, 35 (add 6).

Is 2, 5, 9, 14, ... arithmetic?
No; the differences 3, 4, 5 are not constant.

Find d for 40, 33, 26, ...
$d = 33 - 40 = -7$.

Write the term-to-term rule for 1, 4, 7, 10, ...
Start at 1 and add 3.

Find the nth term of 6, 10, 14, 18, ...
$a_n = 6 + (n - 1)4 = 4n + 2$.

Find the 20th term of 2, 9, 16, ...
$2 + 19 \times 7 = 135$.

Write the first four terms of $a_n = 3n + 5$.
8, 11, 14, 17.

Which term of 4, 9, 14, ... is 99?
$4 + (n - 1)5 = 99$, so $n = 20$.

Find r for 3, 12, 48, ...
$r = 12 \div 3 = 4$.

Find the 5th term of 5, 10, 20, ...
$5 \times 2^4 = 80$.

Find the next term of 243, 81, 27, ...
9 (multiply by 1/3).

Distinguish arithmetic and geometric sequences.
Arithmetic adds a constant difference; geometric multiplies by a constant ratio.

A pattern uses 6, 11, 16, ... tiles. How many in the 8th figure?
$6 + 7 \times 5 = 41$ tiles.

A ball rises 64 cm then half as high each bounce. Height on the 4th bounce?
64, 32, 16, 8 → 8 cm.

Long Questions and Answers

Explain arithmetic sequences with the term-to-term and position-to-term rules.

What is an arithmetic sequence?

A sequence in which the same number (common difference d) is added each time, e.g., 7, 12, 17, 22, ... with d = 5.

What is the term-to-term rule?

It gives the next term from the previous one: 'start at 7 and add 5'.

What is the position-to-term rule?

It gives any term from its position: $a_n = 7 + (n - 1)5 = 5n + 2$.

Use it to find the 30th term.

$5 \times 30 + 2 = 152$.

Derive and use the general term of an arithmetic sequence.

How do the terms build up?

$a_1 = a$, $a_2 = a + d$, $a_3 = a + 2d$, $a_4 = a + 3d$; the number of d's is one less than the position.

What is the general term?

$a_n = a + (n - 1)d$.

Find the 15th term of 11, 8, 5, ...

$a = 11$, $d = -3$: $11 + 14(-3) = -31$.

Which term of 3, 10, 17, ... is 199?

$3 + (n - 1)7 = 199$, so $n - 1 = 28$ and $n = 29$.

Explain geometric sequences with examples.

What is a geometric sequence?

A sequence where each term is the previous term multiplied by a constant ratio r.

How is r found?

Divide any term by the term before it: for 4, 12, 36, ..., r = 3.

What is the nth term?

$a_n = a \cdot r^{n-1}$.

Find the 6th term of 4, 12, 36, ...

$4 \times 3^5 = 972$.

Solve real-life problems involving sequences.

A stadium has 30 seats in the first row and 4 more in each next row. How many seats in row 12?

$30 + 11 \times 4 = 74$.

Ali saves Rs. 200 in week 1, increasing by Rs. 50 each week. Savings in week 10?

$200 + 9 \times 50 = \text{Rs. } 650$.

A culture of 50 bacteria triples every hour. How many after 4 hours?

$50 \times 3^4 = 4{,}050$.

A taxi charges Rs. 150 for the first km and Rs. 40 for each extra km. Fare for 12 km?

$150 + 11 \times 40 = \text{Rs. } 590$.

Explain shape patterns and how to find their rule.

What is a shape pattern?

A series of figures, such as squares of matchsticks, that grows in a regular way.

How is the rule found?

Count the objects in each figure, list the numbers and find the difference or ratio.

Squares in a row need 4, 7, 10, ... matchsticks. Find the nth term.

$d = 3$, so $a_n = 4 + (n - 1)3 = 3n + 1$.

How many matchsticks for 25 squares?

$3 \times 25 + 1 = 76$.

Multiple Choice Questions (MCQs)

The common difference of 9, 13, 17, ... is: (A) 3 (B) 4 (C) 9 (D) 13
Correct answer: (B) 4. 13 – 9.

The next term of 2, 6, 18, 54, ... is: (A) 72 (B) 108 (C) 162 (D) 216
Correct answer: (C) 162. × 3.

The nth term of an arithmetic sequence is: (A) a + nd (B) a + (n – 1)d (C) $a \cdot r^n$ (D) $a \cdot r^{n-1}$
Correct answer: (B) a + (n – 1)d. Standard formula.

The nth term of a geometric sequence is: (A) a + (n – 1)d (B) $a \cdot r^{n-1}$ (C) $a \cdot n^r$ (D) n·r
Correct answer: (B) $a \cdot r^{n-1}$. Standard formula.

Which sequence is arithmetic? (A) 1, 2, 4, 8 (B) 1, 4, 9, 16 (C) 5, 8, 11, 14 (D) 2, 6, 18, 54
Correct answer: (C) 5, 8, 11, 14. d = 3.

Which sequence is geometric? (A) 3, 6, 9, 12 (B) 3, 6, 12, 24 (C) 1, 3, 6, 10 (D) 2, 4, 7, 11
Correct answer: (B) 3, 6, 12, 24. r = 2.

The 10th term of 1, 4, 7, ... is: (A) 25 (B) 28 (C) 31 (D) 30

Correct answer: (B) 28. 1 + 9 × 3.

If $a_n = 2n + 3$, the 5th term is: (A) 10 (B) 13 (C) 15 (D) 8
Correct answer: (B) 13. 2 × 5 + 3.

r for 100, 50, 25, ... is: (A) 2 (B) –50 (C) 1/2 (D) 25
Correct answer: (C) 1/2. 50 ÷ 100.

The term-to-term rule of 10, 7, 4, ... is: (A) Add 3 (B) Subtract 3 (C) Multiply by 3 (D) Divide by 3
Correct answer: (B) Subtract 3. Subtract 3.

The nth term of the odd numbers is: (A) 2n (B) 2n – 1 (C) n + 1 (D) n^2
Correct answer: (B) 2n – 1. 1, 3, 5, ...

1, 4, 9, 16, ... are: (A) Even numbers (B) Square numbers (C) Cube numbers (D) Prime numbers
Correct answer: (B) Square numbers. n^2.

4th term of 3, 6, 12, ... is: (A) 18 (B) 24 (C) 36 (D) 48
Correct answer: (B) 24. 3 × 2^3.

If d is negative, an arithmetic sequence is: (A) Increasing (B) Decreasing (C) Constant (D) Geometric
Correct answer: (B) Decreasing. Terms fall.

Doubling from 5 for 3 steps gives: (A) 15 (B) 30 (C) 40 (D) 20
Correct answer: (C) 40. 5 × 2^3.

Quick Revision Summary

- Sequence: ordered list following a rule; terms a_1, a_2, a_3, ...
- Arithmetic: constant d; $a_n = a + (n – 1)d$.
- Term-to-term: 'start at a, add d'; position-to-term: formula in n.
- Geometric: constant r; $a_n = a \cdot r^{n-1}$.
- Real life: seats, savings, fares (arithmetic); growth, halving (geometric).

Exam Tips

- Always check differences first; if they are not constant, try ratios.
- Write a, d (or r) and n before using a formula.
- To find which term equals a value, set a_n equal to it and solve for n.
- Check your nth-term rule by substituting n = 1 and n = 2.
- In word problems, identify the first term and the fixed change carefully.

Unit 8: Expansion and Factorization

Unit 8 builds the main tools of algebra. It begins with open and closed sentences and polynomials, then shows how to add, subtract, multiply and divide polynomials. Algebraic identities such as $(a + b)^2 = a^2 + 2ab + b^2$ make expansion quick, and factorization reverses expansion by writing an expression as a product.

The unit also covers exponents (powers), the laws of exponents including zero, negative and fractional indices, and standard form / scientific notation for writing very large and very small numbers.

Learning Objectives

- Distinguish open and closed sentences and identify polynomials, their degree and type.
- Add, subtract, multiply and divide polynomials.
- Use algebraic identities to expand expressions and to calculate values like 103^2 and 101×99.
- Factorize expressions by common factors, grouping, identities and splitting the middle term.
- Apply the laws of exponents, including zero, negative and fractional indices.
- Write numbers in standard form and scientific notation.

Key Concepts

8.1 Sentences and Polynomials

A closed sentence is a statement that is clearly true or false, e.g., $5 + 3 = 8$. An open sentence contains a variable and its truth depends on the value, e.g., $x + 3 = 7$.

A polynomial is an algebraic expression whose terms have variables with whole-number exponents only (0, 1, 2, ...), e.g., $3x^2 – 5x + 2$. Expressions such as $1/x + 2$ or $\sqrt{x} + 1$ are not polynomials.

In $7x^2$, 7 is the coefficient and x is the variable; a term without a variable is the constant term. A polynomial may have one variable ($x^2 + 1$), two variables ($x^2y + xy$) or more.

The degree is the highest exponent (or highest sum of exponents in a term). Degree 1: linear; 2: quadratic; 3: cubic; 4: biquadratic. Example: $x^2y + xy$ has degree 3.

8.2 Operations on Polynomials

Add or subtract by combining like terms: $(3x^2 + 2x – 1) + (x^2 – 5x + 4) = 4x^2 – 3x + 3$. When subtracting, change the sign of every term of the second polynomial.

Multiply each term of one polynomial by each term of the other: $(x + 2)(x + 3) = x^2 + 5x + 6$. The degree of the product is the sum of the degrees.

Divide by long division, as with numbers: $(x^2 + 5x + 6) \div (x + 2) = x + 3$ with remainder 0, so $x + 2$ is a factor. If the remainder is 0, the division is exact.

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8.3 Algebraic Identities

$(a + b)^2 = a^2 + 2ab + b^2$; $(a - b)^2 = a^2 - 2ab + b^2$; $(a + b)(a - b) = a^2 - b^2$.

$(a + b)^3 = a^3 + 3ab(a + b) + b^3$; $(a - b)^3 = a^3 - 3ab(a - b) - b^3$.

Using identities for quick calculation: $103^2 = (100 + 3)^2 = 10000 + 600 + 9 = 10609$; $99^2 = (100 - 1)^2 = 9801$; $101 \times 99 = 100^2 - 1^2 = 9999$.

8.4 Factorization

Factorization writes an expression as a product of its factors; it is the reverse of expansion.

Common factor: $6x^2 + 9x = 3x(2x + 3)$. Grouping: $ax + ay + bx + by = a(x + y) + b(x + y) = (a + b)(x + y)$.

Using identities: $x^2 - 49 = (x + 7)(x - 7)$; $x^2 + 10x + 25 = (x + 5)^2$; $a^2 + 2ab + b^2 - c^2 = (a + b)^2 - c^2 = (a + b + c)(a + b - c)$.

Splitting the middle term: for $x^2 + 7x + 12$ find two numbers with product 12 and sum 7 (3 and 4), so $x^2 + 7x + 12 = (x + 3)(x + 4)$. For $3x^2 + 7xy + 2y^2$, split $7xy$ as $6xy + xy$: $(3x + y)(x + 2y)$.

8.5 Exponents and Their Laws

In a^n, a is the base and n is the exponent (index): $2^5 = 32$. 10^5 is read '10 raised to the power 5'.

Laws: $a^m \times a^n = a^{m+n}$; $a^m \div a^n = a^{m-n}$; $(a^m)^n = a^{mn}$; $(ab)^n = a^n b^n$; $(a/b)^n = a^n/b^n$.

Special indices: $a^0 = 1$ ($a \neq 0$); $a^{-n} = 1/a^n$; a^(1/n) is the nth root of a, so 16^(1/2) = 4 and 27^(2/3) = $(\sqrt[3]{27})^2 = 9$.

8.6 Standard Form and Scientific Notation

Scientific notation writes a number as $a \times 10^n$ where $1 \leq a < 10$ and n is an integer: $4{,}500{,}000 = 4.5 \times 10^6$ and $0.00072 = 7.2 \times 10^{-4}$.

Large numbers have a positive power of 10; small numbers (less than 1) have a negative power. The speed of light, about 300,000,000 m/s, is 3×10^8 m/s.

Important Definitions

Closed sentence
A statement that is definitely true or false.

Open sentence
A sentence with a variable whose truth depends on its value.

Polynomial
An expression whose variables have only whole-number exponents.

Coefficient
The number multiplying a variable in a term.

Degree of a polynomial
The highest exponent (or sum of exponents) of any term.

Quadratic polynomial
A polynomial of degree 2.

Cubic polynomial
A polynomial of degree 3.

Biquadratic polynomial
A polynomial of degree 4.

Identity
An equation true for all values of its variables.

Factorization
Writing an expression as a product of factors.

Base and exponent
In a^n, a is the base and n is the exponent.

Scientific notation
Writing a number as $a \times 10^n$ with $1 \le a < 10$.

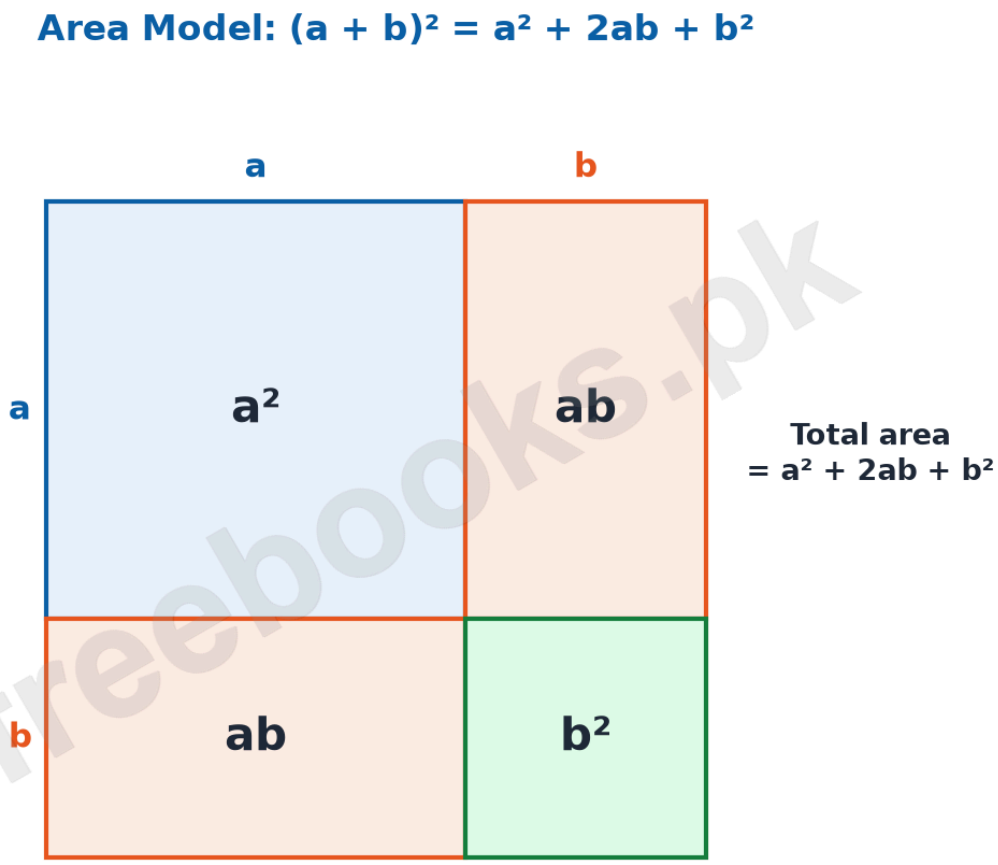
Key Facts at a Glance

Identity / law	Form
$(a + b)^2$	$a^2 + 2ab + b^2$
$(a – b)^2$	$a^2 – 2ab + b^2$
$a^2 – b^2$	$(a + b)(a – b)$
$(a + b)^3$	$a^3 + 3ab(a + b) + b^3$
$(a – b)^3$	$a^3 – 3ab(a – b) – b^3$
$a^m \times a^n$	a^{m+n}
$a^m \div a^n$	a^{m-n}
$(a^m)^n$	a^{mn}
a^0	$1\ (a \neq 0)$
a^{-n}	$1/a^n$

103^2	10609
0.00072	7.2×10^{-4}

Diagrams

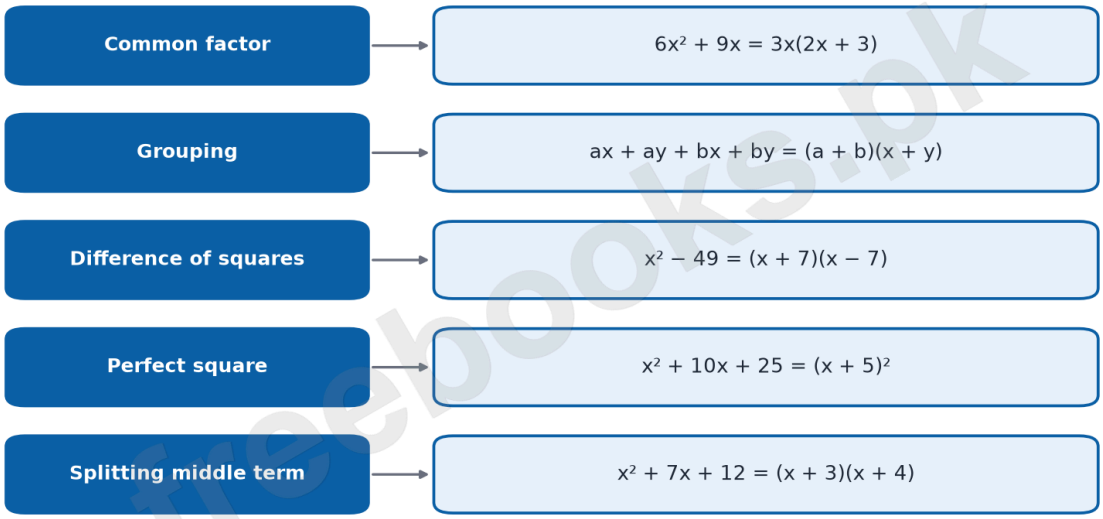
Area model of $(a + b)^2$: A square of side a + b divided into a^2, two ab rectangles and b^2.



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Methods of factorization: A chart of the main factorization methods with an example of each.

Methods of Factorization



Laws of exponents: A chart of the laws of exponents with numerical examples.

Laws of Exponents



Short Questions and Answers

Is x + 5 = 9 an open or closed sentence?
Open, because its truth depends on x.

Is $2x^2 - 3/x$ a polynomial? Why?
No; $3/x = 3x^{-1}$ has a negative exponent.

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Find the degree of $4x^3y + 2xy - 7$.
4 (3 + 1 in the first term).

Subtract $(2a^2 - a + 3)$ from $(5a^2 + 4a - 1)$.
$3a^2 + 5a - 4$.

Multiply $(2x - 1)(x + 4)$.
$2x^2 + 7x - 4$.

Divide $x^2 + 8x + 15$ by $x + 3$.
$x + 5$, remainder 0.

Expand $(2x + 3)^2$.
$4x^2 + 12x + 9$.

Find 98^2 using an identity.
$(100 - 2)^2 = 10000 - 400 + 4 = 9604$.

Find 105×95 using an identity.
$100^2 - 5^2 = 9975$.

Expand $(x + 2)^3$.
$x^3 + 6x^2 + 12x + 8$.

Factorize $9a^2 - 16b^2$.
$(3a + 4b)(3a - 4b)$.

Factorize $x^2 - 5x + 6$.
$(x - 2)(x - 3)$.

Simplify $2^3 \times 2^4 \div 2^5$.
$2^2 = 4$.

Evaluate $5^0 + 2^{-2} + 8\text{^}(1/3)$.
$1 + 1/4 + 2 = 3.25$.

Write 0.0000056 in scientific notation.
5.6×10^{-6}.

Long Questions and Answers

Explain polynomials and their types.

What is a polynomial?

An algebraic expression in which every variable has a whole-number exponent, such as $2x^3 – x + 4$.

How are polynomials named by variables?

In one variable ($x^2 + 1$), two variables ($x^2y – y$) or more ($xyz + 2$).

How are they named by degree?

Linear (1), quadratic (2), cubic (3) and biquadratic (4).

Classify $5x^4 – x^2 + 1$ and $3x – 2$.

The first is biquadratic (degree 4); the second is linear.

Explain the operations on polynomials.

Add $(4x^2 – 3x + 2)$ and $(x^2 + 5x – 7)$.

$5x^2 + 2x – 5$.

Multiply $(x + 3)(x^2 – 2x + 1)$.

$x^3 + x^2 – 5x + 3$.

Divide $x^3 – 1$ by $x – 1$.

$x^2 + x + 1$ with remainder 0.

What is the degree of a product?

The sum of the degrees of the factors: degree 1 × degree 2 gives degree 3.

State and apply the algebraic identities.

Write the square identities.

$(a \pm b)^2 = a^2 \pm 2ab + b^2$ and $(a + b)(a – b) = a^2 – b^2$.

Write the cube identities.

$(a + b)^3 = a^3 + 3ab(a + b) + b^3$ and $(a – b)^3 = a^3 – 3ab(a – b) – b^3$.

Expand $(a – 1)^3$.

$a^3 – 3a(a – 1) – 1 = a^3 – 3a^2 + 3a – 1$.

If $a + b = 7$ and $ab = 10$, find $a^2 + b^2$.

$(a + b)^2 – 2ab = 49 – 20 = 29$.

Explain the methods of factorization with examples.

Taking a common factor

$12x^2y – 18xy^2 = 6xy(2x – 3y)$.

Grouping

$px – py + qx – qy = p(x – y) + q(x – y) = (p + q)(x – y)$.

Using identities

$4x^2 + 12x + 9 = (2x + 3)^2$; $x^2 – 2xy + y^2 – z^2 = (x – y + z)(x – y – z)$.

Splitting the middle term

$2x^2 + 7x + 3$: product 6, sum 7 → 6 and 1: $2x^2 + 6x + x + 3 = (2x + 1)(x + 3)$.

Explain the laws of exponents.

Product and quotient laws

$a^m \times a^n = a^{m+n}$ and $a^m \div a^n = a^{m-n}$; e.g., $3^4 \times 3^2 = 3^6$ and $7^5 \div 7^3 = 49$.

Power laws

$(a^m)^n = a^{mn}$, $(ab)^n = a^n b^n$, $(a/b)^n = a^n/b^n$; e.g., $(3^2)^3 = 729$.

Zero and negative indices

$a^0 = 1$ and $a^{-n} = 1/a^n$; e.g., $(2/3)^{-2} = 9/4$.

Fractional indices

a^(m/n) = $(\sqrt[n]{a})^m$; e.g., 27^(2/3) = 9 and 32^(1/5) = 2.

Explain scientific notation with examples.

What is scientific notation?

Writing a number as $a \times 10^n$ with $1 \le a < 10$ and n an integer.

Write 72,000,000 in scientific notation.

7.2×10^7.

Write 0.000304 in scientific notation.

3.04×10^{-4}.

Write 6.1×10^5 in ordinary form.

610,000.

Multiple Choice Questions (MCQs)

Which is a polynomial? (A) $x + 1/x$ (B) $\sqrt{x} + 2$ (C) $3x^2 – x + 5$ (D) $2/x^2$
Correct answer: (C) $3x^2 – x + 5$. Whole-number exponents.

Degree of $3x^2y^3 – xy$ is: (A) 2 (B) 3 (C) 5 (D) 6
Correct answer: (C) 5. 2 + 3.

A polynomial of degree 3 is: (A) Linear (B) Quadratic (C) Cubic (D) Biquadratic
Correct answer: (C) Cubic. Cubic.

$(x + 4)(x – 4)$ = (A) $x^2 – 16$ (B) $x^2 + 16$ (C) $x^2 – 8x + 16$ (D) $x^2 – 8$

Correct answer: (A) $x^2 - 16$. Difference of squares.

$(a - b)^2$ = (A) $a^2 - b^2$ (B) $a^2 - 2ab + b^2$ (C) $a^2 + 2ab + b^2$ (D) $a^2 + b^2$
Correct answer: (B) $a^2 - 2ab + b^2$. Identity.

101 × 99 = (A) 9999 (B) 10001 (C) 9899 (D) 10099
Correct answer: (A) 9999. $100^2 - 1$.

$x^2 + 6x + 9$ = (A) $(x + 3)^2$ (B) $(x - 3)^2$ (C) $(x + 9)(x + 1)$ (D) $(x + 6)(x + 3)$
Correct answer: (A) $(x + 3)^2$. Perfect square.

Factors of $x^2 + 7x + 10$ are: (A) $(x + 2)(x + 5)$ (B) $(x + 1)(x + 10)$ (C) $(x - 2)(x - 5)$ (D) $(x + 3)(x + 4)$
Correct answer: (A) $(x + 2)(x + 5)$. $2 \times 5 = 10$, $2 + 5 = 7$.

a^0 = (A) 0 (B) 1 (C) a (D) Undefined for $a \neq 0$
Correct answer: (B) 1. Zero index.

2^{-3} = (A) –8 (B) –6 (C) 1/8 (D) 8
Correct answer: (C) 1/8. $1/2^3$.

$(x^3)^4$ = (A) x^7 (B) x^{12} (C) x^{81} (D) x
Correct answer: (B) x^{12}. 3×4.

81^(1/2) = (A) 9 (B) 40.5 (C) 3 (D) 27
Correct answer: (A) 9. $\sqrt{81}$.

4,200 in scientific notation is: (A) 42×10^2 (B) 4.2×10^3 (C) 0.42×10^4 (D) 4.2×10^2
Correct answer: (B) 4.2×10^3. $1 \leq a < 10$.

0.0035 = (A) 3.5×10^{-3} (B) 3.5×10^3 (C) 35×10^{-4} (D) 3.5×10^{-2}
Correct answer: (A) 3.5×10^{-3}. Move point 3 places.

x + 3 = 7 is a(n): (A) Closed sentence (B) Open sentence (C) Identity (D) Polynomial
Correct answer: (B) Open sentence. Has a variable.

Quick Revision Summary

- Closed sentence: true/false; open sentence: contains a variable.
- Polynomial: whole-number exponents; degree → linear, quadratic, cubic, biquadratic.
- Product degree = sum of degrees; exact division leaves remainder 0.
- $(a \pm b)^2 = a^2 \pm 2ab + b^2$; $a^2 - b^2 = (a + b)(a - b)$; $(a \pm b)^3 = a^3 \pm 3ab(a \pm b) \pm b^3$.
- Factorize: common factor, grouping, identities, splitting the middle term.
- Exponent laws; $a^0 = 1$; $a^{-n} = 1/a^n$; a^(1/n) = $\sqrt[n]{a}$.

- Scientific notation: $a \times 10^n$, $1 \leq a < 10$.

Exam Tips

- Always look for a common factor first before trying other methods.
- Check a factorization by multiplying the factors back.
- Write identities at the top of your answer before using them.
- For negative indices, flip the base: $(a/b)^{-n} = (b/a)^n$.
- In scientific notation count how many places the decimal point moves.

Unit 9: Linear Equations and Inequalities

Unit 9 is about straight lines and the equations that describe them. Every straight line (except a vertical one) can be written as y = mx + c, where m is the gradient (slope) and c is the y-intercept. Plotting such lines and finding where two of them meet leads to solving simultaneous linear equations.

These notes cover gradient, horizontal and vertical lines, the slope-intercept form and graphing, ordered pairs, changing the subject of a formula, constructing and solving simultaneous equations by substitution, elimination and graphs, word problems, and solving simple linear inequalities.

Learning Objectives

- Find and interpret the gradient and y-intercept of a straight line.
- Recognise horizontal (y = c) and vertical (x = a) lines.
- Plot graphs of y = mx and y = mx + c using a table of values.
- Change the subject of a formula and substitute values.
- Construct and solve simultaneous linear equations by substitution, elimination and graphs.
- Solve simple linear inequalities and show them on a number line.

Key Concepts

9.1 Gradient and Slope-Intercept Form

The gradient (slope) m of a line through (x_1, y_1) and (x_2, y_2) is $m = (y_2 - y_1)/(x_2 - x_1)$ — the rise divided by the run. For (1, 2) and (3, 8), m = 6/2 = 3.

A positive gradient rises to the right, a negative gradient falls, a horizontal line (y = c) has gradient 0, and a vertical line (x = a) has an undefined gradient.

Slope-intercept form: y = mx + c, where c is the y-intercept (the point (0, c) where the line cuts the y-axis). y = 2x + 1 has gradient 2 and y-intercept 1. y = mx passes through the origin.

9.2 Ordered Pairs and Graphs

An ordered pair (x, y) gives the position of a point; order matters, so (2, 3) ≠ (3, 2).

To draw y = mx + c, choose some x-values, find y from the equation, plot the ordered pairs and join them with a straight line. Likewise, given y, solve the equation to find x.

9.3 Changing the Subject of a Formula

The subject is the variable on its own on one side. To change it, use inverse operations on both sides.

Examples: from v = u + at, t = (v – u)/a; from P = 2(l + w), w = P/2 – l; from $A = \pi r^2$, $r = \sqrt{(A/\pi)}$. Then substitute known values to find the unknown.

9.4 Simultaneous Linear Equations

Two equations in the same two variables that are true at the same time form a pair of simultaneous equations. Their solution is the ordered pair that satisfies both.

Substitution: from $x - y = 4$ write $x = y + 4$ and substitute in $x + y = 10$ to get $2y + 4 = 10$, so $y = 3$ and $x = 7$.

Elimination: add or subtract the equations to remove one variable. From $3x + 2y = 16$ and $x + 2y = 8$, subtracting gives $2x = 8$, so $x = 4$ and $y = 2$.

Graphical method: draw both lines; the point where they cross is the solution. $y = 2x + 1$ and $y = -x + 4$ meet at $(1, 3)$.

9.5 Real-Life Problems

Choose letters for the unknowns, write two equations from the information and solve. Example: 3 pens and 2 books cost Rs. 340, and 1 pen and 2 books cost Rs. 260. Subtracting gives 2 pens = Rs. 80, so a pen costs Rs. 40 and a book Rs. 110.

9.6 Linear Inequalities

An inequality uses $<$, $>$, $\leq$ or $\geq$. It is solved like an equation, except that multiplying or dividing both sides by a negative number reverses the sign.

Examples: $2x + 3 < 11$ gives $x < 4$; $-3x \geq 12$ gives $x \leq -4$; $5 - x > 2$ gives $x < 3$.

On a number line, an open circle shows that the end value is not included ($<$, $>$) and a filled circle shows that it is included ($\leq$, $\geq$).

Important Definitions

Gradient (slope)
The steepness of a line: change in y ÷ change in x.

y-intercept
The point where a line crosses the y-axis, (0, c).

Slope-intercept form
The equation $y = mx + c$ of a straight line.

Ordered pair
A pair (x, y) giving a point's position, where order matters.

Subject of a formula
The variable expressed in terms of the others.

Simultaneous equations
Two equations whose common solution is required.

Substitution method
Solving by replacing one variable with an expression from the other equation.

Elimination method
Solving by adding or subtracting equations to remove a variable.

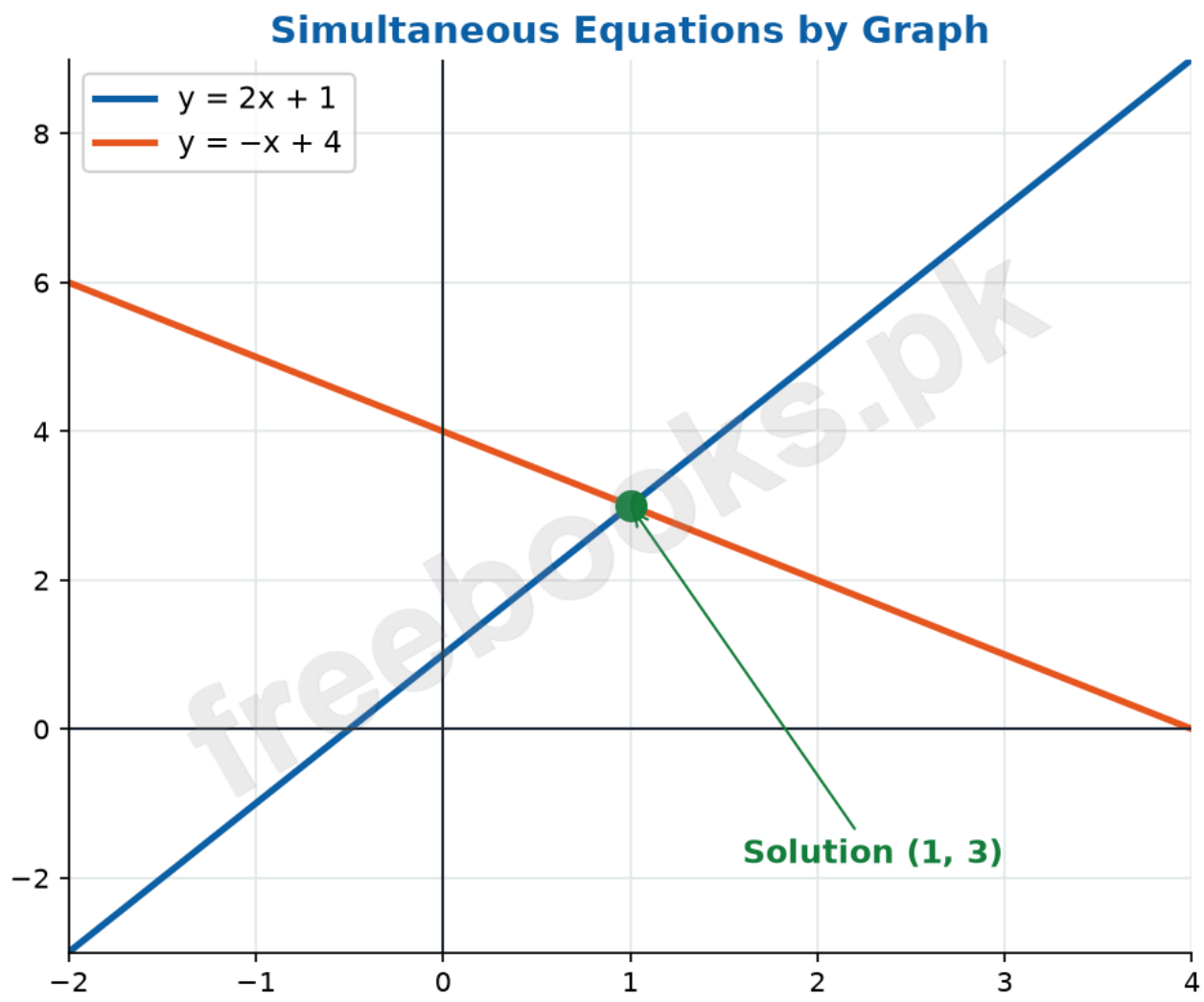
Linear inequality
A statement like $2x + 1 < 7$ using $<$, $>$, $\leq$ or $\geq$.

Key Facts at a Glance

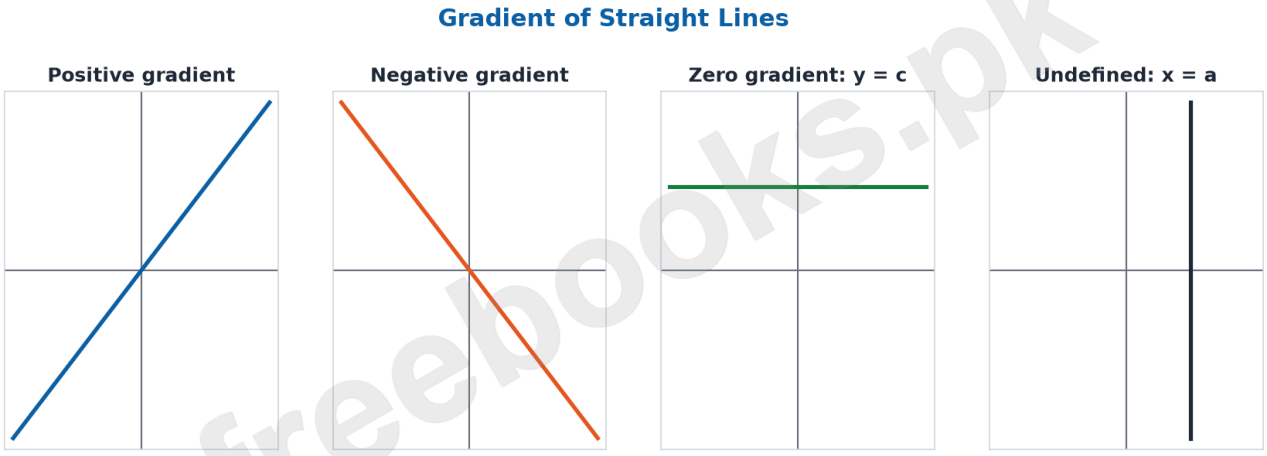
Item	Rule / example
Gradient	$m = (y_2 - y_1)/(x_2 - x_1)$
Slope-intercept form	$y = mx + c$
Horizontal line	$y = c$, gradient 0
Vertical line	$x = a$, gradient undefined
$y = 2x + 1$	$m = 2$, $c = 1$
$v = u + at$	$t = (v - u)/a$
$x + y = 10$, $x - y = 4$	$x = 7$, $y = 3$
$3x + 2y = 16$, $x + 2y = 8$	$x = 4$, $y = 2$
$y = 2x + 1$ and $y = -x + 4$	Meet at (1, 3)
$2x + 3 < 11$	$x < 4$
$-3x \geq 12$	$x \leq -4$
Open / filled circle	Not included / included

Diagrams

Solving simultaneous equations by graph: The lines $y = 2x + 1$ and $y = -x + 4$ crossing at the solution (1, 3).



Gradient of lines: Lines with positive, negative, zero and undefined gradients.

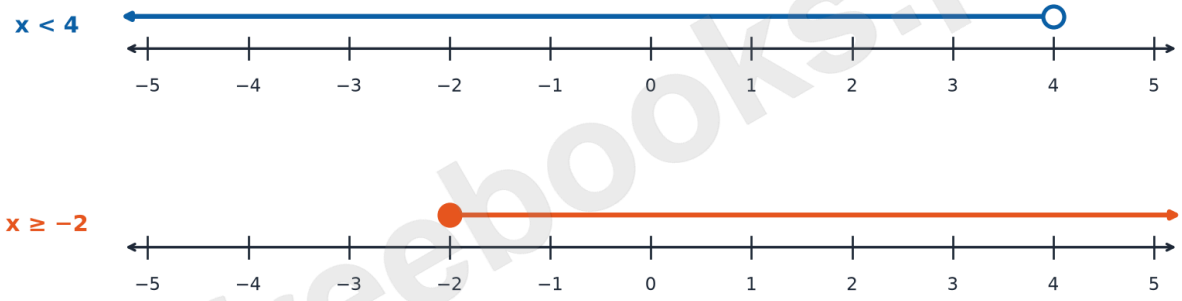


Inequalities on a number line: Number lines showing $x < 4$ and $x \geq -2$ with open and filled circles.

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Inequalities on a Number Line



Open circle: value not included (<, >) Filled circle: value included (≤, ≥)

Short Questions and Answers

Find the gradient of the line through (2, 1) and (6, 9).
(9 – 1)/(6 – 2) = 2.

Write the gradient and y-intercept of y = –3x + 5.
m = –3, c = 5.

What is the gradient of y = 7?
0, because it is a horizontal line.

Write the equation of the vertical line through (4, –1).
x = 4.

Find y when x = 3 in y = 4x – 2.
y = 10.

Find x when y = 11 in y = 2x + 5.
2x = 6, x = 3.

Make r the subject of C = 2πr.
r = C/(2π).

Make b the subject of A = ½bh.
b = 2A/h.

Solve x + y = 12 and x – y = 2.
x = 7, y = 5.

Solve 2x + y = 9 and x – y = 3.

Adding: 3x = 12, x = 4; y = 1.

What does the intersection of two lines represent?
The solution of the pair of simultaneous equations.

The sum of two numbers is 50 and their difference is 12. Find them.
31 and 19.

Solve 4x – 5 ≥ 7.
x ≥ 3.

Solve –2x < 8.
x > –4 (sign reverses).

How is x ≤ 1 shown on a number line?
A filled circle at 1 with an arrow to the left.

Long Questions and Answers

Explain gradient and the slope-intercept form of a line.

What is gradient?

The steepness of a line, found as rise ÷ run = $(y_2 – y_1)/(x_2 – x_1)$.

What do the signs of gradient mean?

Positive rises to the right, negative falls, zero is horizontal and undefined is vertical.

What is y = mx + c?

The slope-intercept form, where m is the gradient and c is the y-intercept.

A line has gradient 3 and passes through (0, –2). Write its equation.

y = 3x – 2.

Draw the graph of y = 2x – 1.

Make a table of values.

x = –1, 0, 1, 2 give y = –3, –1, 1, 3.

Plot the points.

(–1, –3), (0, –1), (1, 1), (2, 3).

Join the points.

They lie on one straight line, which is extended in both directions.

Read the gradient and intercept.

The line rises 2 for every 1 across (m = 2) and cuts the y-axis at –1.

Solve simultaneous equations by substitution and elimination.

Solve 2x + 3y = 12 and x – y = 1 by substitution.

x = 1 + y; 2 + 2y + 3y = 12 → y = 2, x = 3.

Solve 5x + 2y = 19 and 3x – 2y = 5 by elimination.

Adding: 8x = 24 → x = 3; then 2y = 4 → y = 2.

How do you check the answer?

Substitute into both original equations; both must be true.

When is elimination easier?

When one variable has equal or opposite coefficients in both equations.

Solve simultaneous equations graphically.

Draw y = x + 2.

Points (0, 2), (1, 3), (2, 4).

Draw y = –2x + 5.

Points (0, 5), (1, 3), (2, 1).

Find the intersection.

The lines meet at (1, 3).

State the solution.

x = 1, y = 3.

Solve real-life problems using simultaneous equations.

A father is three times as old as his son; together they are 48. Find their ages.

f = 3s and f + s = 48 → 4s = 48, s = 12, f = 36.

2 kg apples and 3 kg bananas cost Rs. 1,100; 1 kg apples and 1 kg bananas cost Rs. 450. Find each price.

From the second, a = 450 – b; 900 – 2b + 3b = 1100 → b = Rs. 200 per kg, a = Rs. 250 per kg.

A rectangle's perimeter is 34 cm and its length is 5 cm more than its width. Find its sides.

2(l + w) = 34, l = w + 5 → w = 6 cm, l = 11 cm.

A purse has Rs. 10 and Rs. 20 notes: 25 notes worth Rs. 350. How many of each?

x + y = 25, 10x + 20y = 350 → y = 10, x = 15.

Explain how to solve linear inequalities.

How is an inequality solved?

Like an equation, using the same operation on both sides.

What is the special rule?

Multiplying or dividing by a negative number reverses the inequality sign.

Solve $3x - 4 > 8$ and $7 - 2x \leq 1$.

$x > 4$; $-2x \leq -6 \rightarrow x \geq 3$.

How are solutions shown on a number line?

Open circle for < or >, filled circle for ≤ or ≥, with an arrow in the direction of the solutions.

Multiple Choice Questions (MCQs)

The gradient of $y = 4x - 3$ is: (A) –3 (B) 4 (C) 3 (D) –4
Correct answer: (B) 4. m = 4.

The y-intercept of $y = 4x - 3$ is: (A) 4 (B) 3 (C) –3 (D) 0
Correct answer: (C) –3. c = –3.

A horizontal line has gradient: (A) 1 (B) 0 (C) Undefined (D) –1
Correct answer: (B) 0. No rise.

$x = 5$ is a: (A) Horizontal line (B) Vertical line (C) Line through origin (D) Curve
Correct answer: (B) Vertical line. Vertical.

The line $y = mx$ passes through: (A) (0, 1) (B) (1, 0) (C) (0, 0) (D) (m, 0)
Correct answer: (C) (0, 0). c = 0.

Gradient through (0, 0) and (2, 6) is: (A) 2 (B) 3 (C) 6 (D) 1/3
Correct answer: (B) 3. 6/2.

If $y = 3x + 1$ and $x = 2$, then $y =$ (A) 5 (B) 6 (C) 7 (D) 8
Correct answer: (C) 7. 6 + 1.

Making h the subject of $V = lbh$ gives: (A) $h = V - lb$ (B) $h = V/(lb)$ (C) $h = lbV$ (D) $h = lb/V$
Correct answer: (B) $h = V/(lb)$. Divide by lb.

Solution of $x + y = 6$, $x - y = 2$ is: (A) (4, 2) (B) (2, 4) (C) (3, 3) (D) (5, 1)
Correct answer: (A) (4, 2). x = 4, y = 2.

The graphical solution is the: (A) y-intercept (B) Intersection point (C) Gradient (D) Origin
Correct answer: (B) Intersection point. Common point.

Solving $2x + 1 = 7$ gives $x =$ (A) 2 (B) 3 (C) 4 (D) 6

Correct answer: (B) 3. 2x = 6.

2x < 10 gives: (A) x < 5 (B) x > 5 (C) x < 20 (D) x ≤ 5
Correct answer: (A) x < 5. Divide by 2.

–x > 3 gives: (A) x > 3 (B) x < –3 (C) x > –3 (D) x < 3
Correct answer: (B) x < –3. Sign reverses.

A filled circle on a number line means: (A) Not included (B) Included (C) Infinity (D) Zero
Correct answer: (B) Included. ≤ or ≥.

(3, 5) and (5, 3) are: (A) Equal (B) Different points (C) Both on y-axis (D) Both on x-axis
Correct answer: (B) Different points. Order matters.

Quick Revision Summary

- $m = (y_2 – y_1)/(x_2 – x_1)$; $y = mx + c$ (m gradient, c y-intercept).
- y = c horizontal (m = 0); x = a vertical (m undefined); y = mx through origin.
- Plot lines from a table of values; ordered pairs (x, y).
- Change subject with inverse operations; then substitute.
- Simultaneous equations: substitution, elimination, graph (intersection point).
- Inequalities: like equations, but × or ÷ by a negative reverses the sign.

Exam Tips

- Label equations (1) and (2) and show which operation you apply.
- Always check simultaneous solutions in both equations.
- Use at least three points when drawing a straight-line graph.
- Underline the sign flip when dividing an inequality by a negative.
- In word problems, state what each letter stands for.

Unit 10: Mensuration

Mensuration is the measurement of lengths, areas and volumes. Unit 10 starts with Pythagoras theorem for right-angled triangles, then moves to the parts of a circle, the length of an arc and the area of a sector.

The second half deals with solid shapes: the surface area and volume of a pyramid, sphere, hemisphere and cone, and of composite solids made by joining these shapes. Unless told otherwise, these notes take $\pi = 22/7$.

Learning Objectives

- State Pythagoras theorem and use it to find a missing side of a right-angled triangle.
- Identify the parts of a circle: radius, diameter, chord, arc, sector, segment and tangent.
- Calculate the length of an arc and the area of a sector.
- Find the surface area and volume of a pyramid.
- Find the surface area and volume of a sphere, hemisphere and cone.
- Solve problems on composite solids.

Key Concepts

10.1 Pythagoras Theorem

In a right-angled triangle, the square of the hypotenuse equals the sum of the squares of the other two sides: $c^2 = a^2 + b^2$, where c is the side opposite the right angle.

Example: legs 6 cm and 8 cm give $c = \sqrt{(36 + 64)} = \sqrt{100} = 10$ cm. To find a leg, subtract: $a = \sqrt{(c^2 – b^2)}$.

Sets of whole numbers that satisfy the theorem are called Pythagorean triples: (3, 4, 5), (5, 12, 13), (8, 15, 17). The converse is also true: if $c^2 = a^2 + b^2$, the triangle is right-angled.

10.2 Circle, Arc and Sector

Radius: centre to the circle. Diameter: a chord through the centre ($d = 2r$). Chord: a segment joining two points on the circle. Arc: a part of the circumference. Sector: the region between two radii and an arc. Segment: the region between a chord and an arc. Tangent: a line touching the circle at one point.

Circumference $= 2\pi r$ and area $= \pi r^2$.

For a central angle x°: arc length $L = (x/360) \times 2\pi r$ and sector area $A = (x/360) \times \pi r^2$. For $r = 7$ cm and $x = 90°$: $L = ¼ \times 44 = 11$ cm and $A = ¼ \times 154 = 38.5$ cm².

10.3 Pyramid

A pyramid has a polygon base and triangular faces meeting at a point called the apex. The height h is the perpendicular from the apex to the base; the slant height l is the height of a triangular face.

Surface area = base area + area of all triangular faces, each face = ½ × base edge × slant height. Volume = ⅓ × base area × h.

Example: square base 12 cm, height 8 cm. Slant height l = $\sqrt{(8^2 + 6^2)}$ = 10 cm. Faces = 4 × ½ × 12 × 10 = 240 cm²; total surface = 240 + 144 = 384 cm². Volume = ⅓ × 144 × 8 = 384 cm³.

10.4 Sphere and Hemisphere

Sphere: surface area = $4\pi r^2$; volume = $(4/3)\pi r^3$. For r = 7 cm: surface = 616 cm² and volume ≈ 1437.33 cm³.

A hemisphere is half a sphere. Curved surface = $2\pi r^2$; total surface (with the flat circle) = $3\pi r^2$; volume = $(2/3)\pi r^3$. For r = 7 cm: curved 308 cm², total 462 cm², volume ≈ 718.67 cm³.

10.5 Cone

A cone has a circular base and an apex. Radius r, height h and slant height l are related by $l^2 = r^2 + h^2$.

Curved surface = πrl; total surface = $\pi r(r + l)$; volume = $\frac{1}{3}\pi r^2 h$.

Example: r = 7 cm, h = 24 cm, so l = 25 cm. Curved surface = 22/7 × 7 × 25 = 550 cm²; total = 22/7 × 7 × 32 = 704 cm²; volume = ⅓ × 22/7 × 49 × 24 = 1232 cm³.

10.6 Composite Solids

A composite solid is made by joining simple solids. Add the volumes of the parts. For surface area, add only the outside surfaces and leave out joined faces.

Example: a cone of radius 7 cm and height 6 cm on a hemisphere of radius 7 cm. Volume = 308 + 718.67 ≈ 1026.67 cm³.

Important Definitions

Hypotenuse
The longest side of a right-angled triangle, opposite the right angle.

Pythagorean triple
Three whole numbers a, b, c with $a^2 + b^2 = c^2$.

Chord
A line segment joining two points on a circle.

Arc
A part of the circumference of a circle.

Sector
The region bounded by two radii and an arc.

Segment
The region bounded by a chord and an arc.

Tangent
A line that touches a circle at exactly one point.

Apex
The top point of a pyramid or cone.

Slant height
The distance from the apex down the slanting surface to the base edge.

Hemisphere
Half of a sphere cut through its centre.

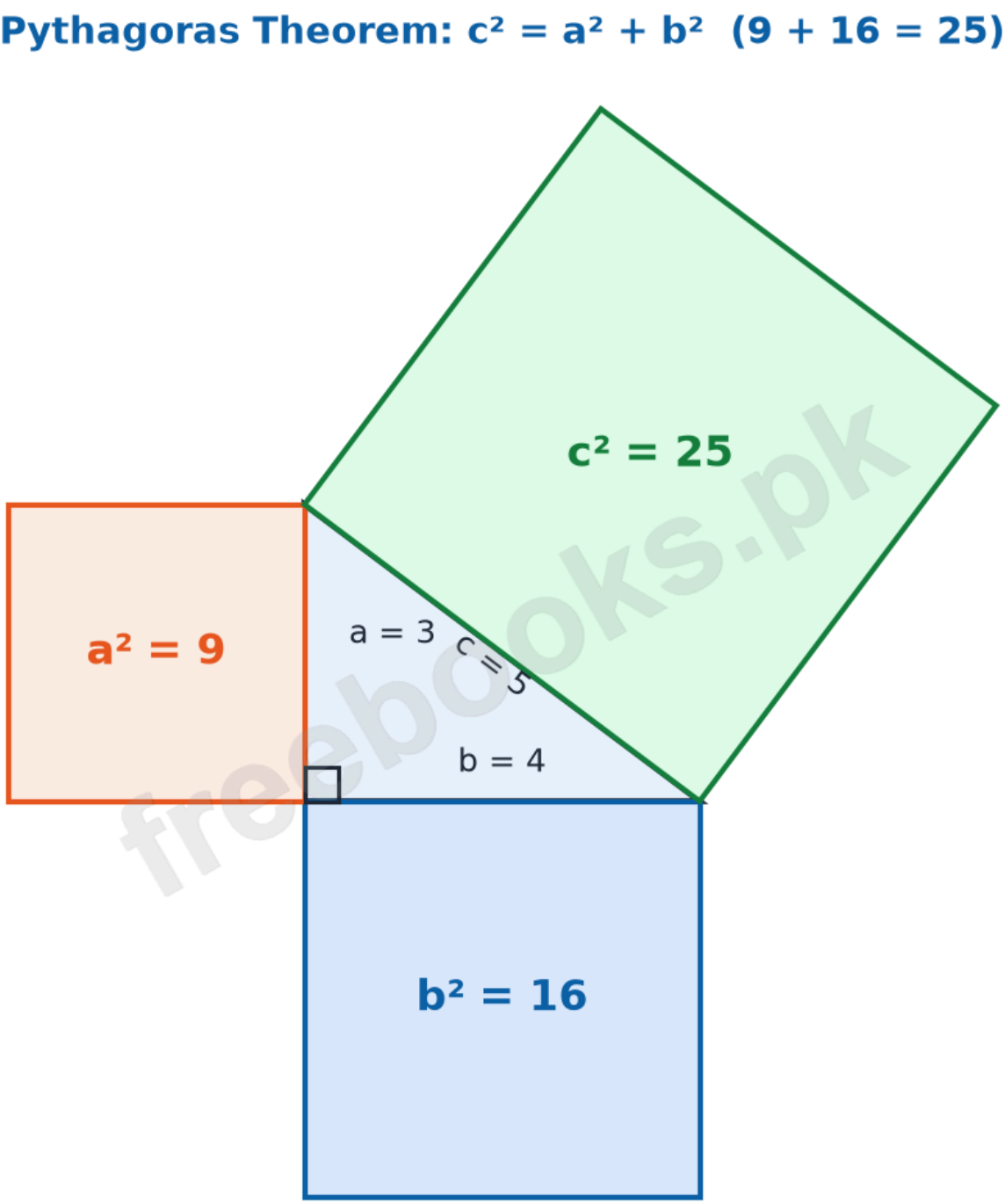
Composite solid
A solid made by combining two or more simple solids.

Key Facts at a Glance

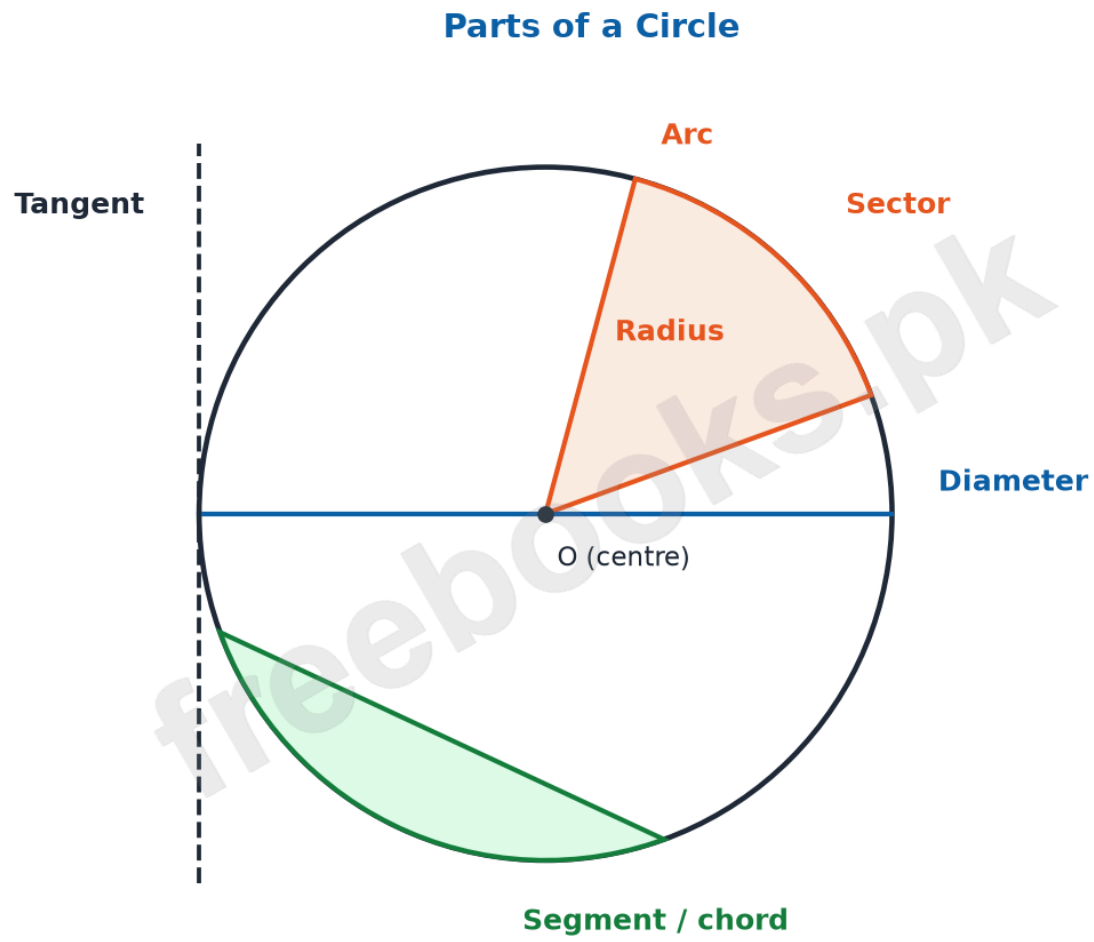
Quantity	Formula
Pythagoras	$c^2 = a^2 + b^2$
Arc length	$(x/360) \times 2\pi r$
Sector area	$(x/360) \times \pi r^2$
Pyramid surface area	Base + Σ ½ × edge × slant height
Pyramid volume	⅓ × base area × h
Sphere surface area	$4\pi r^2$
Sphere volume	$(4/3)\pi r^3$
Hemisphere total surface	$3\pi r^2$
Hemisphere volume	$(2/3)\pi r^3$
Cone slant height	$l^2 = r^2 + h^2$
Cone surfaces	Curved πrl; total $\pi r(r + l)$
Cone volume	$\frac{1}{3}\pi r^2 h$

Diagrams

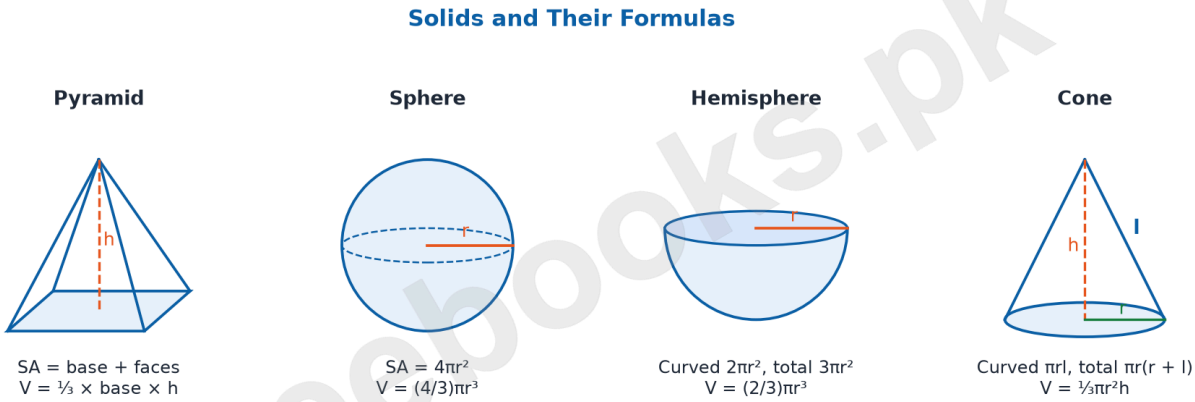
Pythagoras theorem: A right-angled triangle with squares on each side showing $c^2 = a^2 + b^2$.



Parts of a circle: A circle labelled with radius, diameter, chord, arc, sector, segment and tangent.



Solids and formulas: A pyramid, sphere, hemisphere and cone with their surface area and volume formulas.



Short Questions and Answers

State Pythagoras theorem.
In a right-angled triangle, hypotenuse² = sum of squares of the other two sides.

Find the hypotenuse if the legs are 5 cm and 12 cm.
√169 = 13 cm.

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A ladder 17 m long reaches 15 m up a wall. How far is its foot from the wall?
$\sqrt{(289 - 225)} = 8$ m.

Are 7, 24, 25 the sides of a right triangle?
Yes: $49 + 576 = 625$.

What is the difference between a chord and a diameter?
A diameter is a chord that passes through the centre.

Find the arc length for r = 14 cm, x = 45°.
$\frac{1}{8} \times 2 \times 22/7 \times 14 = 11$ cm.

Find the sector area for r = 14 cm, x = 60°.
$\frac{1}{6} \times 22/7 \times 196 \approx 102.67$ cm².

Find the volume of a pyramid with base area 50 cm² and height 9 cm.
$\frac{1}{3} \times 50 \times 9 = 150$ cm³.

Find the slant height of a square pyramid of base 10 cm and height 12 cm.
$\sqrt{(144 + 25)} = 13$ cm.

Find the surface area of a sphere of radius 14 cm.
$4 \times 22/7 \times 196 = 2464$ cm².

Find the volume of a sphere of radius 3 cm (π = 3.14).
$4/3 \times 3.14 \times 27 = 113.04$ cm³.

Find the total surface of a hemisphere of radius 14 cm.
$3 \times 22/7 \times 196 = 1848$ cm².

Find the slant height of a cone with r = 6 cm, h = 8 cm.
10 cm.

Find the curved surface of a cone with r = 7 cm, l = 10 cm.
$22/7 \times 7 \times 10 = 220$ cm².

Find the volume of a cone with r = 3 cm, h = 7 cm.
$\frac{1}{3} \times 22/7 \times 9 \times 7 = 66$ cm³.

Long Questions and Answers

State and apply Pythagoras theorem.

State the theorem.

In a right-angled triangle, $c^2 = a^2 + b^2$, where c is the hypotenuse.

Find the missing leg if c = 26 cm and a = 10 cm.

$b = \sqrt{(676 - 100)} = \sqrt{576} = 24$ cm.

Find the diagonal of a rectangle 9 cm by 12 cm.

$\sqrt{(81 + 144)} = \sqrt{225} = 15$ cm.

What is the converse?

If the square of one side equals the sum of the squares of the other two, the triangle is right-angled.

Explain arc length and sector area.

What fraction of the circle is a sector?

x/360 of the whole circle, where x is the central angle.

Write the formulas.

$L = (x/360) \times 2\pi r$ and $A = (x/360) \times \pi r^2$.

Find L and A for r = 21 cm, x = 120°.

$L = ⅓ \times 132 = 44$ cm; $A = ⅓ \times 1386 = 462$ cm².

Find the perimeter of that sector.

44 + 21 + 21 = 86 cm.

Find the surface area and volume of a pyramid.

Describe a square pyramid.

A square base with four triangular faces meeting at the apex.

Base 16 cm, height 6 cm: find the slant height.

$\sqrt{(6^2 + 8^2)} = 10$ cm.

Find its total surface area.

Faces $4 \times ½ \times 16 \times 10 = 320$ cm²; base 256 cm²; total 576 cm².

Find its volume.

$⅓ \times 256 \times 6 = 512$ cm³.

Explain the sphere and the hemisphere.

Formulas for a sphere

Surface area $4\pi r^2$ and volume $(4/3)\pi r^3$.

Formulas for a hemisphere

Curved $2\pi r^2$, total $3\pi r^2$, volume $(2/3)\pi r^3$.

A ball has radius 21 cm. Find its surface area.

$4 \times 22/7 \times 441 = 5544$ cm^2.

Find the volume of a hemispherical bowl of radius 21 cm.

$2/3 \times 22/7 \times 9261 = 19404$ cm^3.

Find the surface area and volume of a cone.

How are r, h and l related?

$l^2 = r^2 + h^2$, by Pythagoras theorem.

For r = 5 cm and h = 12 cm, find l.

l = 13 cm.

Find its curved and total surface (π = 3.14).

Curved $3.14 \times 5 \times 13 = 204.1$ cm^2; total $3.14 \times 5 \times 18 = 282.6$ cm^2.

Find its volume.

$⅓ \times 3.14 \times 25 \times 12 = 314$ cm^3.

Solve a problem on a composite solid.

Describe the solid.

A toy is a cone of radius 7 cm and height 24 cm standing on a hemisphere of the same radius.

Volume of the cone

$⅓ \times 22/7 \times 49 \times 24 = 1232$ cm^3.

Volume of the hemisphere

$2/3 \times 22/7 \times 343 \approx 718.67$ cm^3.

Total volume and outer surface

Volume ≈ 1950.67 cm^3; surface = cone curved 550 + hemisphere curved 308 = 858 cm^2.

Multiple Choice Questions (MCQs)

In a right triangle, the longest side is the: (A) Base (B) Perpendicular (C) Hypotenuse (D) Median

Correct answer: (C) Hypotenuse. Opposite the right angle.

Legs 9 and 12 give hypotenuse: (A) 13 (B) 15 (C) 21 (D) 17

Correct answer: (B) 15. √225.

Which is a Pythagorean triple? (A) (2, 3, 4) (B) (4, 5, 6) (C) (8, 15, 17) (D) (6, 7, 8)

Correct answer: (C) (8, 15, 17). 64 + 225 = 289.

A chord through the centre is a: (A) Radius (B) Diameter (C) Tangent (D) Arc
Correct answer: (B) Diameter. Longest chord.

Region between two radii and an arc: (A) Segment (B) Sector (C) Chord (D) Tangent
Correct answer: (B) Sector. Sector.

Arc length formula is: (A) (x/360)πr² (B) (x/360)2πr (C) 2πr² (D) πr/x
Correct answer: (B) (x/360)2πr. Fraction of circumference.

Volume of a pyramid: (A) Base × h (B) ½ × base × h (C) ⅓ × base × h (D) ¼ × base × h
Correct answer: (C) ⅓ × base × h. One third.

Surface area of a sphere: (A) 2πr² (B) 3πr² (C) 4πr² (D) πr²
Correct answer: (C) 4πr². 4πr².

Volume of a sphere: (A) (4/3)πr³ (B) (2/3)πr³ (C) ⅓πr³ (D) 4πr³
Correct answer: (A) (4/3)πr³. (4/3)πr³.

Total surface of a hemisphere: (A) 2πr² (B) 3πr² (C) 4πr² (D) πr²
Correct answer: (B) 3πr². 2πr² + πr².

Curved surface of a cone: (A) πrh (B) πrl (C) 2πrl (D) πr²
Correct answer: (B) πrl. πrl.

Volume of a cone: (A) πr²h (B) ⅓πr²h (C) ½πr²h (D) (4/3)πr²h
Correct answer: (B) ⅓πr²h. One third of cylinder.

Cone with r = 3, h = 4 has slant height: (A) 5 (B) 7 (C) 6 (D) 12
Correct answer: (A) 5. √25.

Sector area for x = 180° is: (A) πr² (B) ½πr² (C) ¼πr² (D) 2πr²
Correct answer: (B) ½πr². Half the circle.

A line touching a circle at one point is a: (A) Chord (B) Secant (C) Tangent (D) Radius
Correct answer: (C) Tangent. Tangent.

Quick Revision Summary

- $c^2 = a^2 + b^2$; triples (3,4,5), (5,12,13), (8,15,17).
- Arc L = (x/360)2πr; sector A = (x/360)πr².
- Pyramid: SA = base + triangular faces; V = ⅓ × base × h; slant from Pythagoras.
- Sphere 4πr², (4/3)πr³; hemisphere 2πr² / 3πr², (2/3)πr³.
- Cone $l^2 = r^2 + h^2$; curved πrl; total πr(r + l); V = ⅓πr²h.
- Composite: add volumes; count only outer surfaces.

Exam Tips

- Draw and label a sketch before using any formula.
- Check whether the question uses $\pi = 22/7$ or 3.14.
- Use the slant height for surface area and the vertical height for volume.
- Write units: cm for length, cm^2 for area, cm^3 for volume.
- Choose r as a multiple of 7 when estimating with 22/7 to keep working simple.

Unit 11: Congruency, Similarity and Transformation

Unit 11 compares shapes and moves them. Two figures are congruent when they have exactly the same shape and size, and similar when they have the same shape but possibly a different size. Simple conditions such as SSS, SAS, ASA and RHS tell us when two triangles are congruent.

The unit also covers geometric constructions with a ruler and compasses — triangles, quadrilaterals and the bisectors of lines and angles — and three transformations on the coordinate plane: translation, rotation and enlargement.

Learning Objectives

- Recognise congruent (≅) and similar (~) figures.
- Use the SSS, SAS, ASA and RHS conditions for congruent triangles.
- Use equal ratios of corresponding sides for similar figures.
- Construct triangles, quadrilaterals, perpendicular bisectors and angle bisectors.
- Translate and rotate shapes (90° and 180° about the origin) and find the centre of rotation.
- Enlarge shapes with a positive or negative scale factor and find the centre of enlargement.

Key Concepts

11.1 Congruent Figures

Congruent figures have the same shape and the same size; one fits exactly on the other. The symbol is ≅. Corresponding sides and corresponding angles are equal.

Two triangles are congruent if: SSS — all three sides are equal; SAS — two sides and the included angle are equal; ASA — two angles and the included side are equal; RHS — both are right-angled with equal hypotenuse and one other equal side.

Three equal angles (AAA) are not enough for congruence; they only show that the triangles are similar.

11.2 Similar Figures

Similar figures have the same shape but may differ in size. The symbol is ~. Corresponding angles are equal and corresponding sides are in the same ratio.

If △ABC ~ △PQR with AB = 4, PQ = 6, so the ratio is 2 : 3 and the scale factor is 1.5. If BC = 5, then QR = 7.5.

All congruent figures are similar (ratio 1 : 1), but similar figures are not always congruent.

11.3 Constructions

Triangle with three sides (SSS): draw one side as the base, draw arcs from its ends with radii

equal to the other two sides, and join the base ends to the point where the arcs cross. This works only if any two sides together are longer than the third.

Perpendicular bisector of AB: with radius more than half of AB, draw arcs from A and B above and below the line; join the two crossing points. The line cuts AB at its midpoint at 90°.

Angle bisector: from the vertex draw an arc cutting both arms; from these two points draw equal arcs that cross; join the vertex to the crossing point. It divides the angle into two equal parts.

Quadrilaterals are constructed by splitting them into triangles, e.g., drawing a diagonal first and then constructing the two triangles on it.

11.4 Translation

A translation slides every point of a shape the same distance in the same direction. The image is congruent to the object and faces the same way.

A translation of a units right and b units up maps (x, y) to (x + a, y + b). For example, translating (2, 1) by 3 right and 4 down gives (5, –3).

11.5 Rotation

A rotation turns a shape about a fixed point (the centre of rotation) through an angle, clockwise or anticlockwise. The image is congruent to the object.

About the origin: 90° anticlockwise (x, y) → (–y, x); 90° clockwise (x, y) → (y, –x); 180° (x, y) → (–x, –y). So (3, 1) becomes (–1, 3), (1, –3) and (–3, –1).

To find the centre of rotation, join two pairs of corresponding points and draw the perpendicular bisector of each join; they meet at the centre.

11.6 Enlargement

An enlargement changes the size of a shape using a centre and a scale factor k. Each image point lies on the line from the centre through the object point, with image distance = k × object distance. About the origin, (x, y) → (kx, ky).

k > 1 makes the shape bigger; 0 < k < 1 makes it smaller; a negative k puts the image on the opposite side of the centre and turns it upside down. The image is similar to the object.

To find the centre of enlargement, draw lines through pairs of corresponding points; they meet at the centre.

Important Definitions

Congruent figures
Figures with the same shape and size (≅).

Similar figures
Figures with the same shape, equal angles and proportional sides (~).

Included angle

The angle between two given sides.

Scale factor
The ratio of a length in the image to the matching length in the object.

Perpendicular bisector
A line cutting a segment at its midpoint at right angles.

Angle bisector
A ray dividing an angle into two equal angles.

Transformation
A change in the position, orientation or size of a shape.

Translation
A slide of a shape without turning or resizing.

Rotation
A turn of a shape about a fixed centre through an angle.

Enlargement
A change in size about a centre by a scale factor.

Object and image
The shape before and after a transformation.

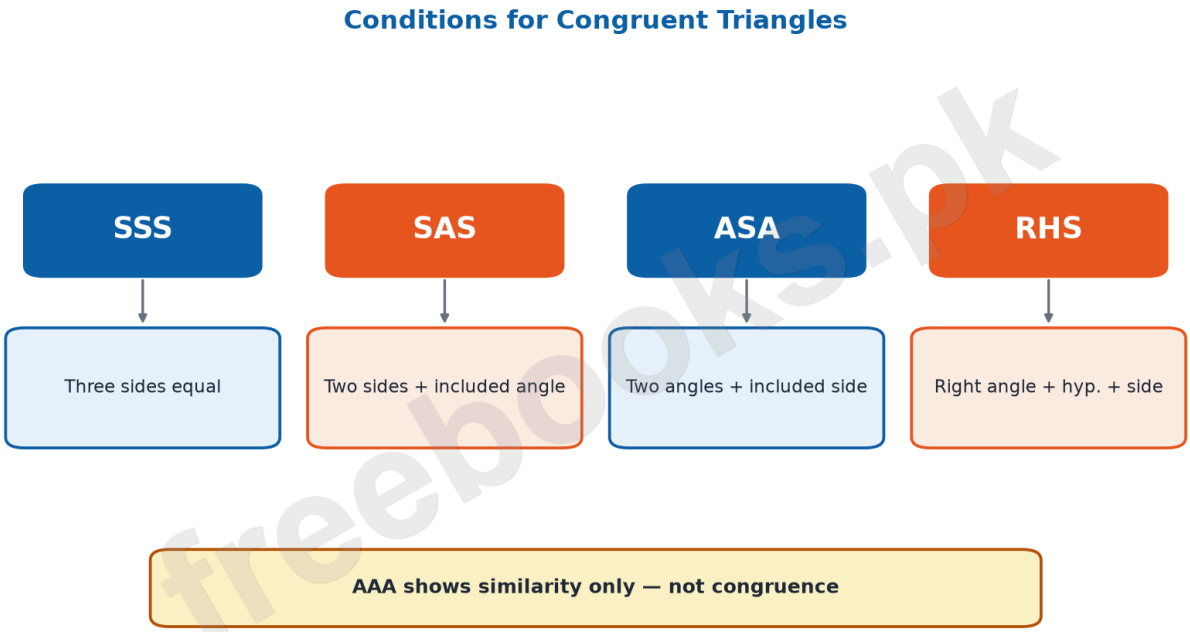
Key Facts at a Glance

Rule	Result
Congruence conditions	SSS, SAS, ASA, RHS
AAA	Similar only, not congruent
Similar figures	Equal angles, sides in same ratio
Translation (a, b)	$(x, y) \to (x + a, y + b)$
90° anticlockwise	$(x, y) \to (-y, x)$
90° clockwise	$(x, y) \to (y, -x)$
180° rotation	$(x, y) \to (-x, -y)$
Enlargement about O, k	$(x, y) \to (kx, ky)$
$k > 1$	Image larger

$0 < k < 1$	Image smaller
$k < 0$	Image on opposite side, inverted
Translation / rotation image	Congruent to object

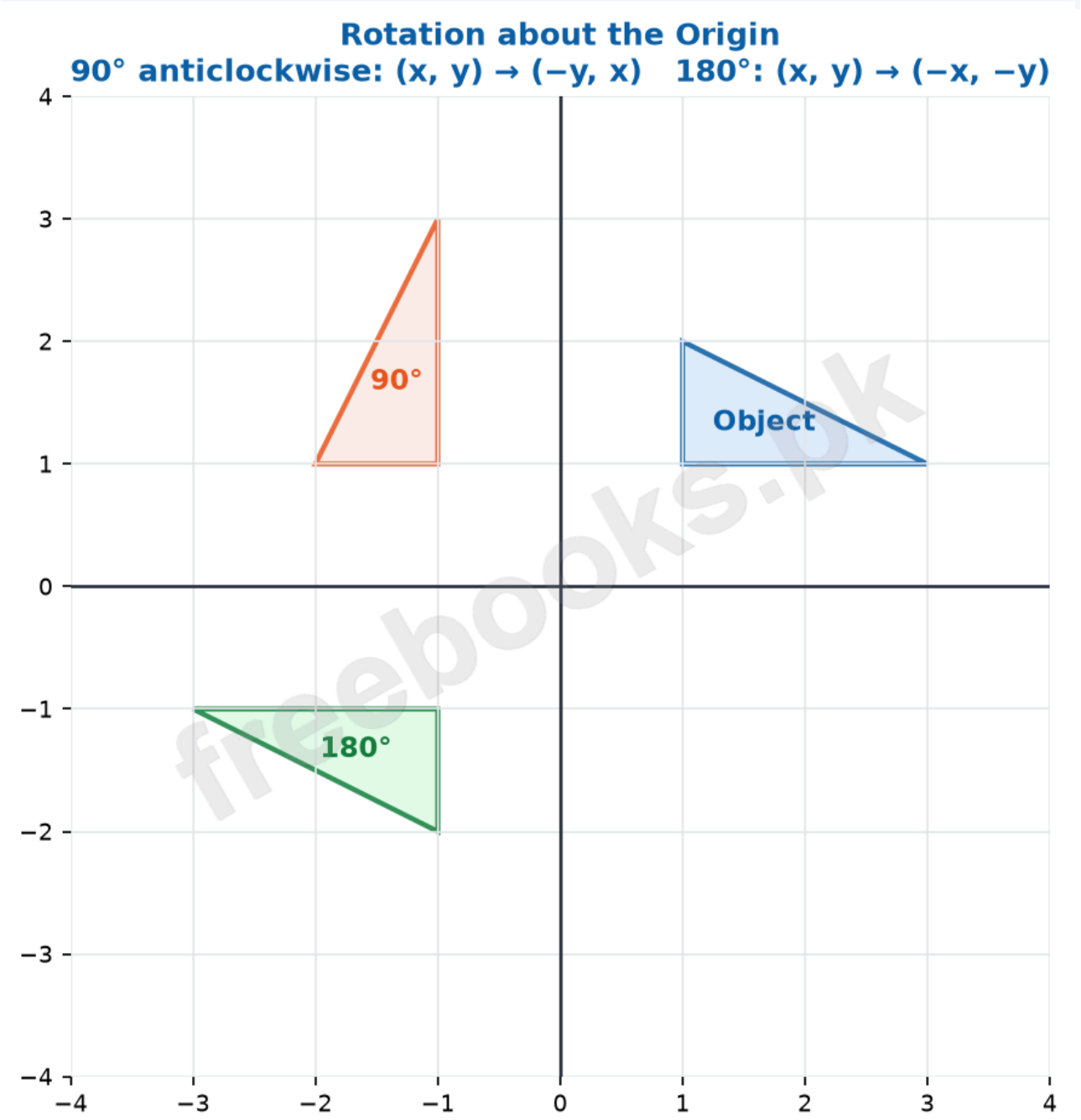
Diagrams

Congruence conditions: Chart showing the SSS, SAS, ASA and RHS conditions for congruent triangles.



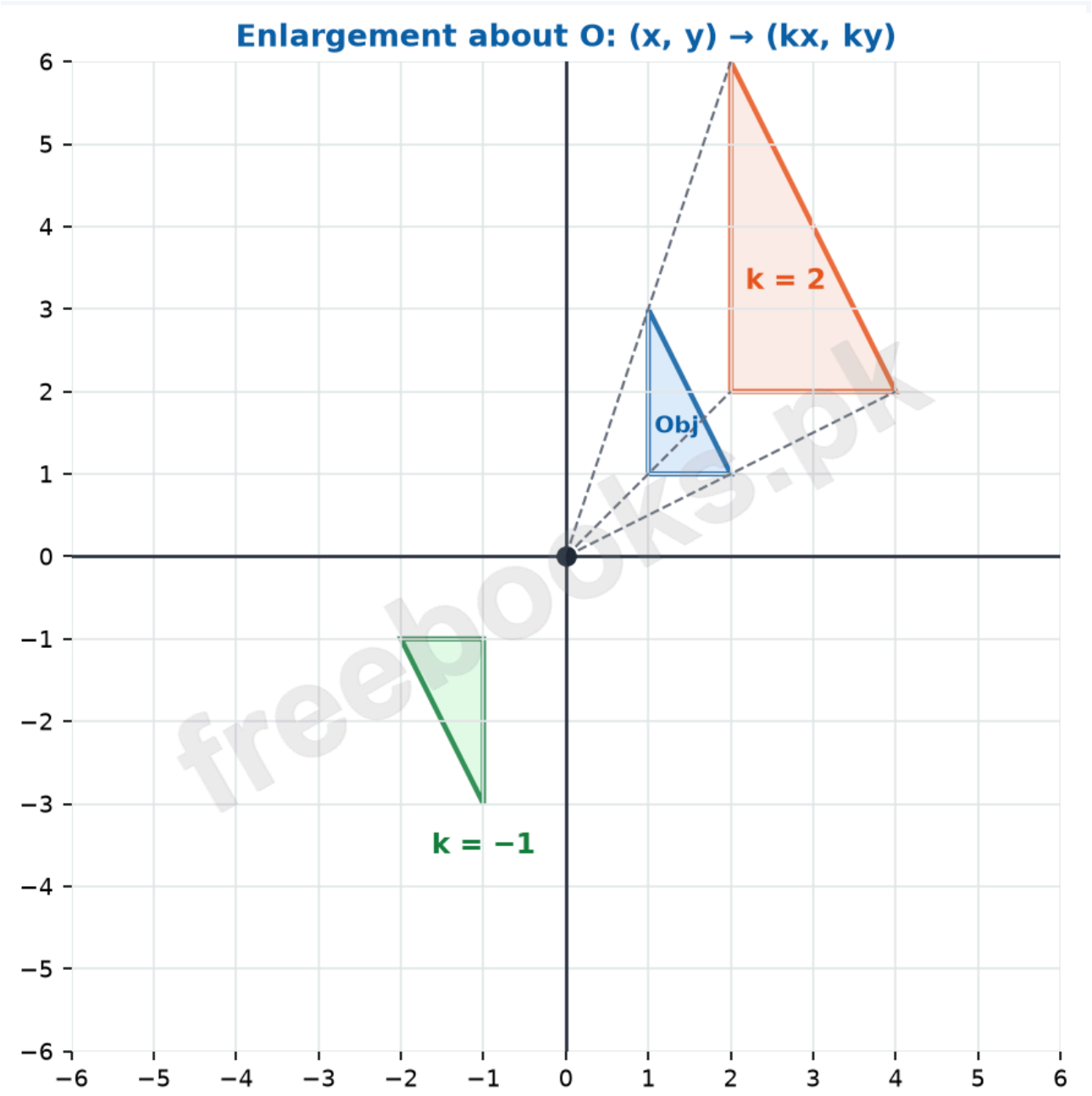
Rotation about the origin: A triangle rotated 90° anticlockwise and 180° about the origin on a grid.

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Enlargement: A triangle enlarged by scale factor 2 and −1 about the origin.

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Short Questions and Answers

What is the difference between congruent and similar figures?
Congruent: same shape and size; similar: same shape, size may differ.

Name the four conditions for congruent triangles.
SSS, SAS, ASA and RHS.

Why is AAA not a congruence condition?
Triangles with equal angles can have different sizes.

△ABC ~ △DEF, AB = 3, DE = 9, BC = 4. Find EF.
Ratio 1 : 3, so EF = 12.

Can a triangle have sides 3 cm, 4 cm and 8 cm?
No, because 3 + 4 < 8.

What does a perpendicular bisector do?
It cuts a segment into two equal parts at 90°.

Translate (–2, 5) by 4 right and 3 down.
(2, 2).

Rotate (4, 2) through 90° anticlockwise about O.
(–2, 4).

Rotate (4, 2) through 90° clockwise about O.
(2, –4).

Rotate (–3, 1) through 180° about O.
(3, –1).

Enlarge (2, –1) by scale factor 3 about O.
(6, –3).

Enlarge (4, 6) by scale factor ½ about O.
(2, 3).

Enlarge (1, 2) by scale factor –2 about O.
(–2, –4).

Is the image of a rotation congruent to the object?
Yes; only position and direction change.

How do you find a centre of enlargement?
Join corresponding points with lines; they meet at the centre.

Long Questions and Answers

Explain the conditions for congruent triangles.

SSS

If all three sides of one triangle equal the three sides of another, the triangles are congruent.

SAS

Two sides and the angle between them are equal in both triangles.

ASA

Two angles and the side between them are equal in both triangles.

RHS

Both triangles have a right angle, equal hypotenuses and one other pair of equal sides.

Explain similar figures with an example.

What makes figures similar?

Equal corresponding angles and corresponding sides in the same ratio.

How is scale factor found?

Divide a side of the image by the matching side of the object.

Two similar rectangles are 4 × 6 and 10 × x. Find x.

Scale factor 10/4 = 2.5, so x = 15.

How are congruence and similarity related?

Congruence is the special case of similarity with scale factor 1.

Describe how to construct a triangle and bisectors.

Construct △ABC with AB = 6 cm, BC = 5 cm, CA = 4 cm.

Draw AB = 6 cm; arc of 5 cm from B and 4 cm from A; join A and B to the crossing point C.

Bisect AB.

With equal radius more than 3 cm, draw arcs from A and B on both sides; join the crossing points.

Bisect angle A.

Arc from A cutting both arms; equal arcs from those two points; join A to their crossing point.

How do you check?

Measure: the two halves of AB and of angle A should be equal.

Explain translation and rotation.

Translation rule

Moving a right and b up maps (x, y) to (x + a, y + b).

Translate P(1, 1), Q(3, 1), R(1, 2) by 2 left and 3 up.

P′(–1, 4), Q′(1, 4), R′(–1, 5).

Rotate P, Q, R through 90° anticlockwise about O.

P′(–1, 1), Q′(–1, 3), R′(–2, 1).

Rotate P, Q, R through 180° about O.

P′(–1, –1), Q′(–3, –1), R′(–1, –2).

Explain enlargement with positive and negative scale factors.

What does an enlargement need?

A centre of enlargement and a scale factor.

Enlarge A(1, 1), B(2, 1), C(1, 3) by k = 2 about O.

A′(2, 2), B′(4, 2), C′(2, 6); sides are doubled.

Enlarge the same triangle by k = –1 about O.

A′(–1, –1), B′(–2, –1), C′(–1, –3); same size, opposite side, upside down.

Is the image congruent or similar?

Similar for any k; congruent only when k = 1 or –1.

How are centres of rotation and enlargement found?

Centre of rotation

Join two pairs of corresponding points and draw the perpendicular bisectors of these joins; they cross at the centre.

Centre of enlargement

Draw straight lines through each object point and its image; they all meet at the centre.

Find the scale factor after locating the centre.

Divide the distance from the centre to an image point by the distance to the matching object point.

When is the centre between object and image?

When the scale factor is negative.

Multiple Choice Questions (MCQs)

Symbol for congruent is: (A) ~ (B) ≅ (C) = (D) ≈
Correct answer: (B) ≅. ≅.

Symbol for similar is: (A) ~ (B) ≅ (C) ∥ (D) ⊥
Correct answer: (A) ~. ~.

Which is NOT a congruence condition? (A) SSS (B) SAS (C) AAA (D) RHS
Correct answer: (C) AAA. AAA gives similarity.

RHS applies to: (A) Any triangle (B) Right-angled triangles (C) Equilateral triangles (D) Quadrilaterals
Correct answer: (B) Right-angled triangles. Right angle needed.

Similar figures always have equal: (A) Sides (B) Areas (C) Angles (D) Perimeters

Correct answer: (C) Angles. Shape kept.

If the scale factor is 1, figures are: (A) Congruent (B) Not similar (C) Larger (D) Smaller
Correct answer: (A) Congruent. Same size.

A perpendicular bisector meets the segment at: (A) 45° (B) 60° (C) 90° (D) 180°
Correct answer: (C) 90°. Right angle.

(2, 3) translated 1 right, 2 up: (A) (3, 5) (B) (1, 1) (C) (4, 4) (D) (3, 1)
Correct answer: (A) (3, 5). Add.

(2, 3) rotated 180° about O: (A) (3, 2) (B) (–2, –3) (C) (–3, 2) (D) (2, –3)
Correct answer: (B) (–2, –3). Negate both.

(2, 3) rotated 90° anticlockwise about O: (A) (–3, 2) (B) (3, –2) (C) (–2, 3) (D) (3, 2)
Correct answer: (A) (–3, 2). (–y, x).

(2, 3) rotated 90° clockwise about O: (A) (–3, 2) (B) (3, –2) (C) (–2, –3) (D) (2, –3)
Correct answer: (B) (3, –2). (y, –x).

Enlargement by k = 3 about O maps (1, 2) to: (A) (3, 6) (B) (4, 5) (C) (1, 6) (D) (3, 2)
Correct answer: (A) (3, 6). Multiply.

A scale factor of ½ makes the image: (A) Larger (B) Smaller (C) Same size (D) Inverted
Correct answer: (B) Smaller. Half.

A negative scale factor places the image: (A) On the same side (B) On the opposite side of the centre (C) At the centre (D) Nowhere
Correct answer: (B) On the opposite side of the centre. Through the centre.

Translation changes a shape's: (A) Size (B) Shape (C) Position only (D) Angles
Correct answer: (C) Position only. Slide.

Quick Revision Summary

- Congruent ≅: same shape and size; similar ~: same shape, proportional sides.
- Congruence tests: SSS, SAS, ASA, RHS (not AAA).
- Constructions: SSS triangle with arcs; perpendicular and angle bisectors.
- Translation (x + a, y + b).
- Rotation about O: 90° acw (–y, x); 90° cw (y, –x); 180° (–x, –y).
- Enlargement about O: (kx, ky); k < 0 → opposite side, inverted.

Exam Tips

- Write corresponding vertices in the same order: $\triangle ABC \cong \triangle PQR$ means $A \leftrightarrow P$, $B \leftrightarrow Q$, $C \leftrightarrow R$.
- Keep construction arcs visible; examiners look for them.
- Label image points with a dash: $A \to A'$.
- Check rotations by turning the page with your hand.
- For enlargement, always measure distances from the centre, not from the shape.

Unit 12: Information Handling

Information handling is about collecting, organising, displaying and interpreting data. Unit 12 explains the types of data, frequency distributions with class limits, class boundaries and cumulative frequency, and how to show data in bar graphs, pie charts and histograms.

It then covers measures of central tendency — mean, median and mode — for ungrouped and grouped data, and measures of spread — variance and standard deviation — which show how far the data are scattered around the mean.

Learning Objectives

- Distinguish discrete and continuous data, and grouped and ungrouped data.
- Make frequency tables with class limits, class boundaries, class marks and cumulative frequency.
- Draw and read bar graphs, pie charts, histograms and cumulative frequency graphs.
- Find the mean, median and mode of ungrouped and grouped data.
- Find the variance and standard deviation of ungrouped data.
- Interpret averages and spread in real-life situations.

Key Concepts

12.1 Types of Data

Discrete data can take only separate values, usually counted (number of students, goals). Continuous data can take any value in a range, usually measured (height, mass, time).

Ungrouped (raw) data are listed one by one. Grouped data are arranged in classes such as 10–19, 20–29 with a frequency for each class.

12.2 Frequency Distribution

Class limits are the end values of a class (lower 20, upper 29 in 20–29). Class boundaries close the gaps: subtract 0.5 from the lower limit and add 0.5 to the upper, giving 19.5–29.5. The class size (width) is the difference between boundaries, here 10.

The class mark (midpoint) is (lower limit + upper limit)/2 = 24.5.

Cumulative frequency (c.f.) is the running total of frequencies. It tells how many values lie below each upper boundary and is used for the median and for a cumulative frequency graph (ogive).

12.3 Graphs

Bar graph: separate bars of equal width for categories or discrete data. Histogram: touching bars drawn on class boundaries for continuous grouped data.

Pie chart: a circle divided into sectors; angle of a sector = (frequency ÷ total) × 360°. If 15 of 60 students like Science, its angle is 90°.

Cumulative frequency graph: plot c.f. against upper class boundaries and join with a smooth curve or line segments.

12.4 Mean, Median and Mode

Ungrouped data: mean = $\Sigma x/n$; median = middle value after arranging in order (average of the two middle values if n is even); mode = most frequent value.

Grouped data: mean = $\Sigma fx/\Sigma f$, where x is the class mark.

Median = $l + (h/f)(n/2 - c)$, where l is the lower boundary of the median class, h the class size, f its frequency and c the cumulative frequency before it.

Mode = $l + [(fm - f1)/((fm - f1) + (fm - f2))] \times h$, where fm is the frequency of the modal class and f1, f2 are the frequencies of the classes before and after it.

12.5 Worked Example (Grouped Data)

Marks of 30 students: 10–19 (4), 20–29 (6), 30–39 (10), 40–49 (7), 50–59 (3). Class marks 14.5, 24.5, 34.5, 44.5, 54.5; $\Sigma fx = 1025$, so mean = $1025/30 \approx 34.17$.

c.f.: 4, 10, 20, 27, 30. $n/2 = 15$ lies in 30–39, so median = $29.5 + (10/10)(15 - 10) = 34.5$.

Modal class 30–39 ($fm = 10$, $f1 = 6$, $f2 = 7$): mode = $29.5 + [4/(4 + 3)] \times 10 \approx 35.21$.

12.6 Variance and Standard Deviation

Variance $S^2 = \Sigma x^2/n - (\Sigma x/n)^2$; standard deviation $S = \sqrt{(\text{variance})}$. A larger S means the data are more spread out.

For 4, 6, 8, 10, 12: $\Sigma x = 40$, $\Sigma x^2 = 360$, $n = 5$. $S^2 = 72 - 64 = 8$ and $S = \sqrt{8} \approx 2.83$.

Standard deviation is never negative. If all values are equal (e.g., 5, 5, 5), the variance and standard deviation are 0.

Important Definitions

Discrete data
Data with separate countable values.

Continuous data
Data that can take any value in a range.

Frequency
The number of times a value or class occurs.

Class boundaries
Limits adjusted by 0.5 so that classes have no gaps.

Class mark
The midpoint of a class.

Cumulative frequency
The running total of frequencies.

Histogram
A graph of touching bars for grouped continuous data.

Mean
The sum of values divided by their number.

Median
The middle value of ordered data.

Mode
The most frequent value.

Variance
The mean of squares minus the square of the mean.

Standard deviation
The positive square root of variance.

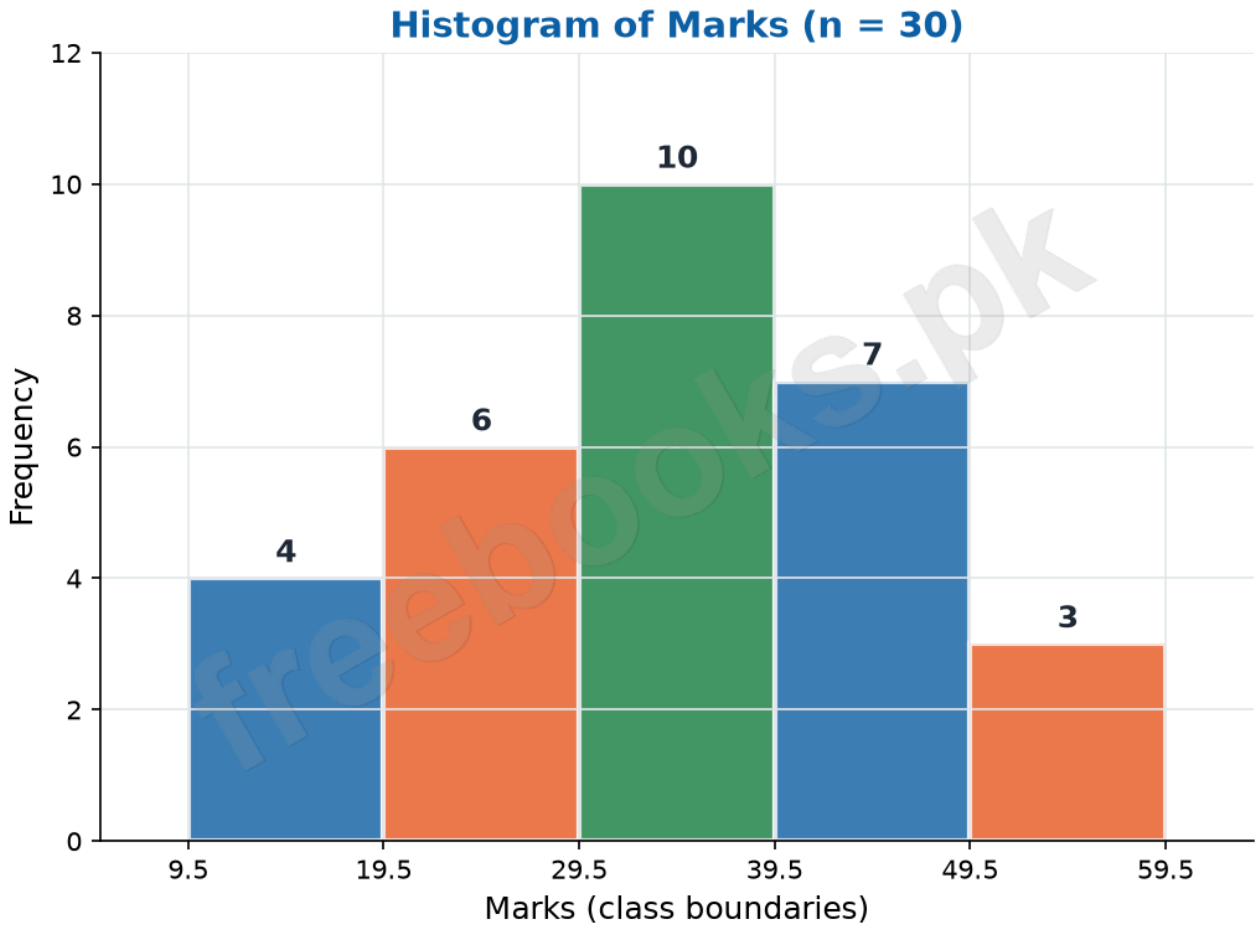
Key Facts at a Glance

Measure	Formula / result
Class boundaries of 20–29	19.5–29.5
Class mark of 20–29	24.5
Pie chart angle	(f ÷ total) × 360°
Mean (grouped)	$\Sigma fx / \Sigma f$
Median (grouped)	$l + (h/f)(n/2 - c)$
Mode (grouped)	$l + (fm - f1)/((fm - f1) + (fm - f2)) \times h$
Variance	$\Sigma x^2/n - (\Sigma x/n)^2$
Standard deviation	√variance
Worked mean	≈ 34.17
Worked median	34.5
Worked mode	≈ 35.21

SD of 4, 6, 8, 10, 12	$\sqrt{8} \approx 2.83$

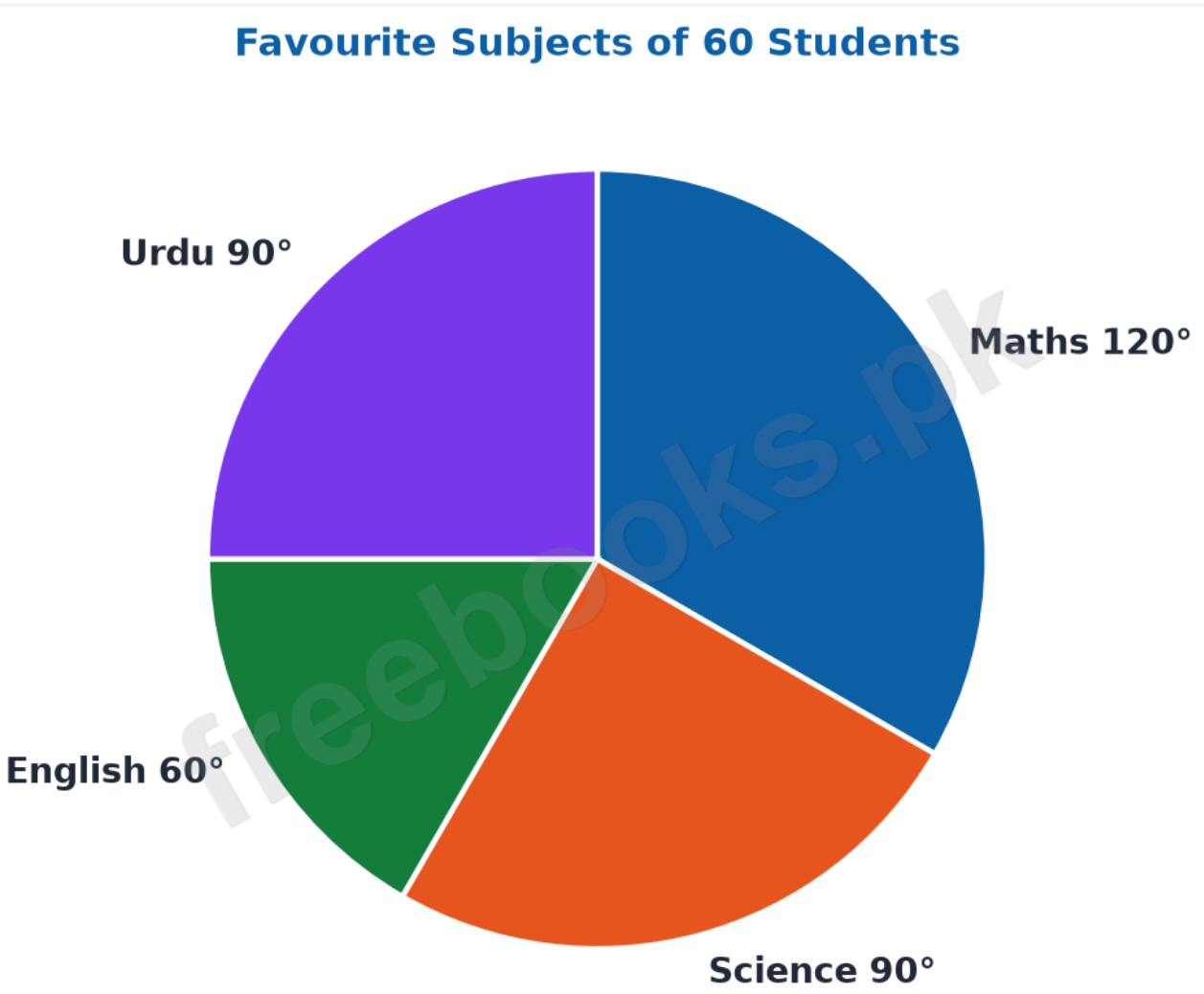
Diagrams

Histogram: Histogram of the marks of 30 students drawn on class boundaries.

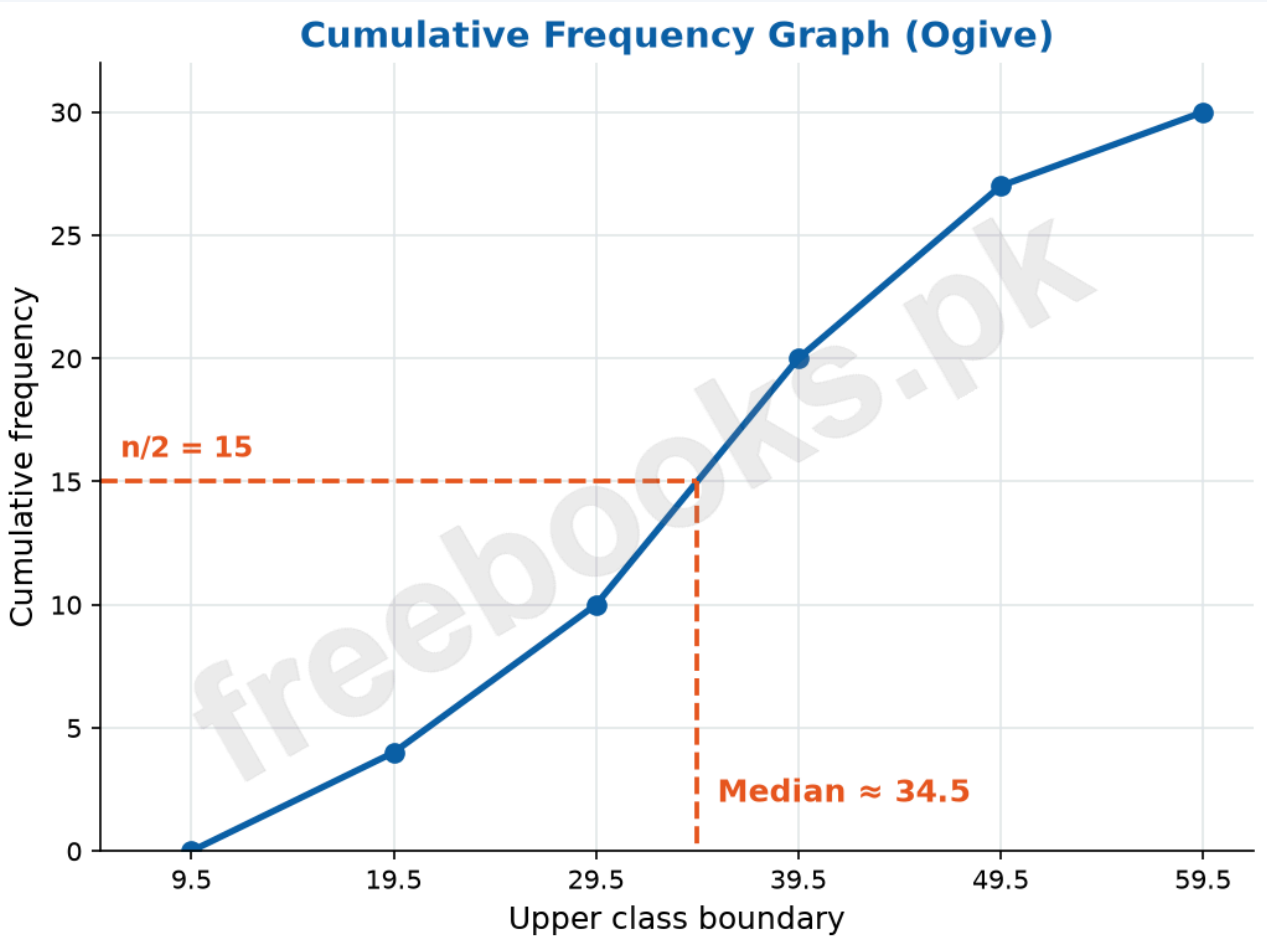


Pie chart: Pie chart of the favourite subjects of 60 students with sector angles.

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Cumulative frequency graph: Ogive of the marks data with the median read at n/2 = 15.



Short Questions and Answers

Is the number of cars in a car park discrete or continuous?
Discrete.

Is the mass of a baby discrete or continuous?
Continuous.

Write the class boundaries of 40–49.
39.5–49.5.

Find the class mark of 40–49.
44.5.

What is cumulative frequency used for?
Finding the median and drawing an ogive.

12 out of 48 people chose tea. Find the pie-chart angle.
(12/48) × 360° = 90°.

Find the mean of 3, 5, 7, 9, 11.

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35/5 = 7.

Find the median of 8, 3, 5, 9, 2.
Ordered 2, 3, 5, 8, 9 → 5.

Find the median of 4, 7, 9, 12.
(7 + 9)/2 = 8.

Find the mode of 2, 4, 4, 5, 7, 4.
4.

How does a histogram differ from a bar graph?
Histogram bars touch and are drawn on class boundaries.

Find the variance of 1, 2, 3, 4, 5.
55/5 – 9 = 2.

Find the standard deviation of 1, 2, 3, 4, 5.
$\sqrt{2} \approx 1.41$.

What is the variance of 5, 5, 5, 5?
0, since all values are equal.

Can standard deviation be negative?
No; it is a square root, so it is 0 or positive.

Long Questions and Answers

Explain types of data and frequency distributions.

Discrete vs continuous

Discrete data are counted and take separate values; continuous data are measured and can take any value in a range.

Grouped vs ungrouped

Ungrouped data list each value; grouped data place values in classes with frequencies.

Class limits, boundaries and class marks

For 30–39: limits 30 and 39, boundaries 29.5–39.5, class mark 34.5, class size 10.

Cumulative frequency

For frequencies 4, 6, 10, 7, 3 the c.f. are 4, 10, 20, 27, 30.

Draw a pie chart for the favourite subjects of 60 students.

Data

Maths 20, Science 15, English 10, Urdu 15.

Angles

Maths 120°, Science 90°, English 60°, Urdu 90° (total 360°).

Drawing

Draw a circle and a radius; measure each angle with a protractor in turn and label each sector.

Reading

The largest sector, Maths, is the most popular subject.

Find the mean of the grouped marks data.

Data

10–19 (4), 20–29 (6), 30–39 (10), 40–49 (7), 50–59 (3); n = 30.

Class marks

14.5, 24.5, 34.5, 44.5, 54.5.

fx values

58, 147, 345, 311.5, 163.5; $\Sigma fx = 1025$.

Mean

1025/30 ≈ 34.17 marks.

Find the median and mode of the grouped marks data.

Median class

n/2 = 15; c.f. 4, 10, 20 → median class 30–39.

Median

29.5 + (10/10)(15 – 10) = 34.5.

Modal class

30–39, with fm = 10, f1 = 6, f2 = 7.

Mode

29.5 + [4/(4 + 3)] × 10 ≈ 35.21.

Find the variance and standard deviation of 2, 4, 6, 8.

Σx and mean

$\Sigma x = 20$, n = 4, mean = 5.

Σx^2

4 + 16 + 36 + 64 = 120.

Variance

$120/4 - 5^2 = 30 - 25 = 5$.

Standard deviation

$\sqrt{5} \approx 2.24$.

Compare the spread of two data sets.

Set A: 6, 6, 6, 6

Mean 6, standard deviation 0 — no spread.

Set B: 3, 5, 7, 9

Mean 6; $\Sigma x^2 = 164$; variance $41 - 36 = 5$; SD ≈ 2.24.

Conclusion

Both have the same mean, but B is more spread out.

Why is spread useful?

An average alone does not show how consistent the data are; SD measures consistency.

Multiple Choice Questions (MCQs)

Heights of students are: (A) Discrete (B) Continuous (C) Qualitative (D) Ungrouped only
Correct answer: (B) Continuous. Measured.

Class boundaries of 10–19: (A) 10–19 (B) 9.5–19.5 (C) 9–20 (D) 10.5–19.5
Correct answer: (B) 9.5–19.5. ±0.5.

Class mark of 10–19: (A) 14 (B) 14.5 (C) 15 (D) 29
Correct answer: (B) 14.5. (10 + 19)/2.

Bars touch in a: (A) Bar graph (B) Pie chart (C) Histogram (D) Line graph
Correct answer: (C) Histogram. Continuous data.

A sector for 25% of the data has angle: (A) 25° (B) 45° (C) 90° (D) 180°
Correct answer: (C) 90°. ¼ × 360°.

Mean of 2, 4, 6, 8: (A) 4 (B) 5 (C) 6 (D) 20
Correct answer: (B) 5. 20/4.

Median of 1, 3, 3, 6, 7: (A) 3 (B) 4 (C) 6 (D) 3.5
Correct answer: (A) 3. Middle value.

Mode of 1, 2, 2, 3, 3, 3: (A) 1 (B) 2 (C) 3 (D) 2.5
Correct answer: (C) 3. Most frequent.

Mean of grouped data: (A) $\Sigma f/n$ (B) $\Sigma fx/\Sigma f$ (C) $\Sigma x/f$ (D) Σfx^2
Correct answer: (B) $\Sigma fx/\Sigma f$. Class marks used.

In the median formula, c is the: (A) Class size (B) c.f. before the median class (C) Frequency of median class (D) Lower limit
Correct answer: (B) c.f. before the median class. Preceding c.f.

In the mode formula, fm is the frequency of the: (A) First class (B) Median class (C) Modal class (D) Last class
Correct answer: (C) Modal class. Modal class.

Variance S^2 = (A) $\Sigma x/n$ (B) $\Sigma x^2/n - (\Sigma x/n)^2$ (C) $(\Sigma x)^2$ (D) $\sqrt{\Sigma x}$
Correct answer: (B) $\Sigma x^2/n - (\Sigma x/n)^2$. Formula.

SD of 7, 7, 7: (A) 7 (B) 0 (C) 1 (D) 49
Correct answer: (B) 0. No spread.

Standard deviation is the square root of: (A) Mean (B) Median (C) Variance (D) Mode
Correct answer: (C) Variance. $\sqrt{S^2}$.

An ogive is drawn using: (A) Frequency (B) Cumulative frequency (C) Class marks (D) Angles
Correct answer: (B) Cumulative frequency. c.f. vs boundary.

Quick Revision Summary

- Discrete (counted) vs continuous (measured); grouped vs ungrouped.
- Boundaries = limits $\mp$ 0.5; class mark = midpoint; c.f. = running total.
- Pie angle = (f/total) × 360°; histogram bars touch.
- Mean $\Sigma fx/\Sigma f$; median $l + (h/f)(n/2 - c)$; mode $l + (fm - f1)/((fm - f1) + (fm - f2)) \times h$.
- Variance $\Sigma x^2/n - (\Sigma x/n)^2$; SD = $\sqrt{\text{variance}} \geq 0$.

Exam Tips

- Make a full table (x, f, fx, c.f.) before using any grouped formula.
- Use class boundaries, not limits, for l in the median and mode formulas.
- Check that pie-chart angles add up to 360°.
- Arrange ungrouped data in order before finding the median.
- Round only the final answer, not intermediate steps.

Unit 13: Probability

Probability measures how likely an event is to happen, on a scale from 0 (impossible) to 1 (certain). Unit 13 explains the language of chance, the difference between experimental and theoretical probability, and the complement rule P(not A) = 1 – P(A).

It then deals with combined events: mutually exclusive events and the addition (OR) rule, independent events and the multiplication (AND) rule, and listing outcomes with sample-space tables and tree diagrams.

Learning Objectives

- Describe events as impossible, unlikely, even chance, likely or certain, and place them on a probability scale.
- Find theoretical probability using P(E) = favourable outcomes ÷ total outcomes.
- Find experimental probability (relative frequency) from trials and expected frequency.
- Use P(not A) = 1 – P(A).
- Apply the addition rule to mutually exclusive events and the multiplication rule to independent events.
- List outcomes of combined events using tables and tree diagrams.

Key Concepts

13.1 Language and Scale of Probability

An experiment (such as tossing a coin) has possible results called outcomes; the set of all outcomes is the sample space. An event is one or more outcomes.

Probability lies between 0 and 1: 0 means impossible (getting 7 on an ordinary die), 1 means certain (the sun rising tomorrow), ½ means even chance (head on a fair coin). Values near 0 are unlikely and values near 1 are likely.

13.2 Theoretical Probability

When all outcomes are equally likely, P(E) = number of favourable outcomes ÷ total number of outcomes.

For one fair die: P(even) = 3/6 = ½; P(number greater than 4) = 2/6 = ⅓. For a bag of 3 red and 5 blue balls: P(red) = 3/8.

Complement rule: P(not A) = 1 – P(A). If P(rain) = 0.3, then P(no rain) = 0.7. The probabilities of all outcomes add up to 1.

13.3 Experimental Probability

Experimental probability (relative frequency) = number of times the event happened ÷ number of trials. If a coin shows heads 28 times in 50 tosses, the experimental probability of a head is 28/50 =

0.56.

As the number of trials increases, experimental probability usually gets closer to the theoretical value.

Expected frequency = probability × number of trials. A fair die rolled 120 times is expected to show six about 120 × ⅙ = 20 times.

13.4 Mutually Exclusive Events (OR rule)

Events are mutually exclusive if they cannot happen together, so P(A and B) = 0. Getting 1 and getting 6 on one roll of a die are mutually exclusive.

Addition rule: P(A or B) = P(A) + P(B). So P(1 or 6) = ⅙ + ⅙ = ⅓.

13.5 Independent Events (AND rule)

Events are independent if one does not affect the other, such as tossing a coin and rolling a die.

Multiplication rule: P(A and B) = P(A) × P(B). P(head and six) = ½ × ⅙ = 1/12. Drawing a red ball twice with replacement from 3 red and 5 blue: (3/8) × (3/8) = 9/64.

13.6 Combined Events: Tables and Tree Diagrams

A tree diagram shows each stage as branches labelled with probabilities. Multiply along a path to get the probability of that outcome, and add the paths for 'or'. Two coin tosses give HH, HT, TH, TT, each ¼; P(at least one head) = ¾.

A sample-space table lists all outcomes of two dice: 36 equally likely pairs. P(sum = 7) = 6/36 = ⅙; P(a double) = 6/36 = ⅙; P(sum = 12) = 1/36.

Important Definitions

Experiment
An action with uncertain results, such as rolling a die.

Outcome
One possible result of an experiment.

Sample space
The set of all possible outcomes.

Event
One or more outcomes of an experiment.

Equally likely outcomes
Outcomes that have the same chance of happening.

Theoretical probability

Favourable outcomes ÷ total outcomes.

Experimental probability
Number of successes ÷ number of trials.

Complementary event
The event 'not A'; P(not A) = 1 – P(A).

Mutually exclusive events
Events that cannot happen at the same time.

Independent events
Events where one does not affect the other.

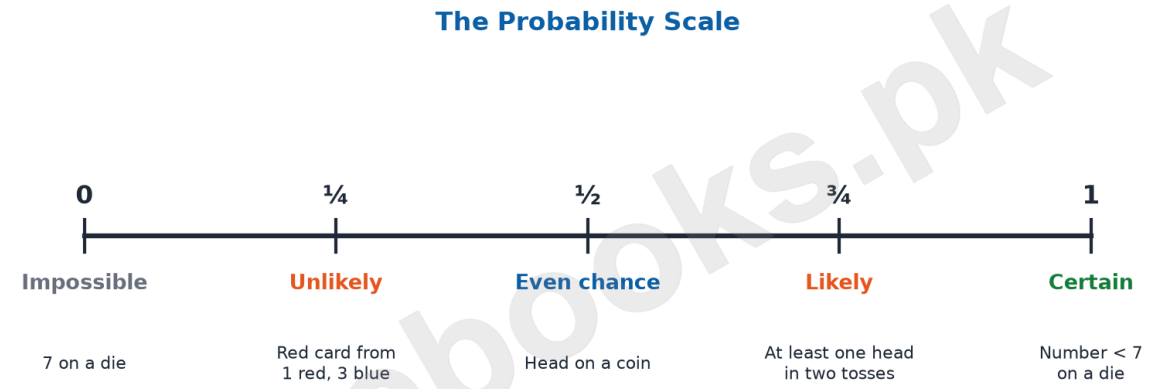
Tree diagram
A branching diagram showing outcomes of combined events.

Key Facts at a Glance

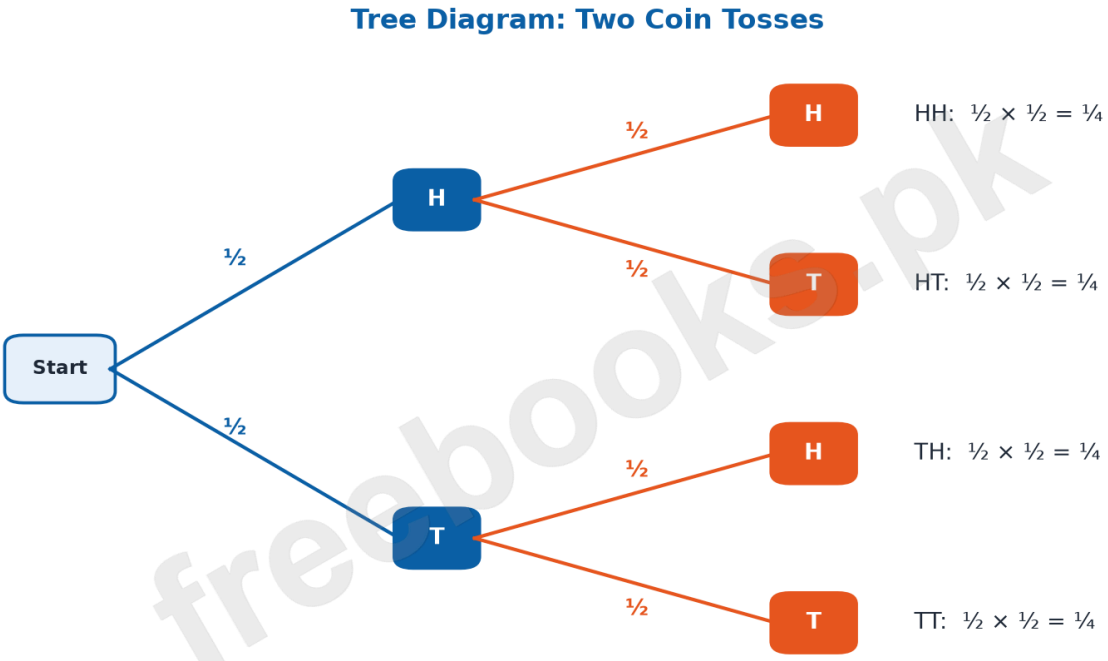
Rule / case	Result
Probability range	$0 \le P(E) \le 1$
Theoretical probability	Favourable ÷ total
Complement	P(not A) = 1 – P(A)
Mutually exclusive (OR)	P(A or B) = P(A) + P(B)
Independent (AND)	P(A and B) = P(A) × P(B)
Expected frequency	P × number of trials
P(even) on a die	½
P(head and six)	1/12
Two coins: P(at least one head)	¾
Two dice: P(sum 7)	⅙
Two dice: P(sum 12)	1/36
28 heads in 50 tosses	0.56

Diagrams

Probability scale: A scale from 0 to 1 marked impossible, unlikely, even chance, likely and certain with examples.



Tree diagram for two coins: A tree diagram of two coin tosses with probabilities on each branch.



Sample space of two dice: A 6 × 6 table of sums of two dice with the six ways of scoring 7 highlighted.

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Two Dice: 36 Outcomes — P(sum = 7) = 6/36 = ⅙

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

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Short Questions and Answers

What is the probability of an impossible event?
0.

Give an example of a certain event.
Getting a number less than 7 on an ordinary die.

Find P(5) for a fair die.
⅙.

Find P(prime) for a fair die.

2, 3, 5 → 3/6 = ½.

A bag has 4 green and 6 yellow balls. Find P(green).
4/10 = 2/5.

If P(win) = 0.35, find P(not win).
0.65.

A spinner landed on red 18 times in 60 spins. Find the experimental probability.
18/60 = 0.3.

A fair coin is tossed 200 times. How many heads are expected?
100.

What are mutually exclusive events?
Events that cannot happen at the same time.

Find P(2 or 3) on one roll of a die.
⅙ + ⅙ = ⅓.

What are independent events?
Events where the result of one does not affect the other.

Find P(tail and even number) for a coin and a die.
½ × ½ = ¼.

Two coins are tossed. Find P(two tails).
¼.

Two dice are rolled. How many outcomes are there?
36.

Two dice are rolled. Find P(a double).
6/36 = ⅙.

Long Questions and Answers

Explain the probability scale.

What range does probability have?

From 0 to 1, written as fractions, decimals or percentages.

What do 0 and 1 mean?

0 means impossible and 1 means certain.

What does ½ mean?

Even chance, like a head on a fair coin.

Place 'a day of the week starting with S' on the scale.

Saturday and Sunday: 2/7 ≈ 0.29, which is unlikely.

Find theoretical probabilities for a fair die.

List the sample space.

{1, 2, 3, 4, 5, 6}, six equally likely outcomes.

P(odd)

1, 3, 5 → 3/6 = ½.

P(multiple of 3)

3, 6 → 2/6 = ⅓.

P(not a multiple of 3)

1 – ⅓ = ⅔.

Compare experimental and theoretical probability.

Definitions

Experimental = successes ÷ trials; theoretical = favourable ÷ total outcomes.

Example

A die rolled 60 times showed four 14 times: experimental P(4) = 14/60 ≈ 0.23; theoretical ⅙ ≈ 0.17.

Why do they differ?

Results of a small number of trials vary by chance.

What happens with more trials?

The experimental value usually comes closer to the theoretical value.

Explain mutually exclusive events and the addition rule.

Meaning

Two events that cannot occur together; P(A and B) = 0.

Addition rule

P(A or B) = P(A) + P(B).

A card is picked from cards numbered 1–10. Find P(less than 3 or greater than 8).

2/10 + 2/10 = 2/5.

Are 'even' and 'greater than 4' on a die mutually exclusive?

No; 6 is in both, so the simple addition rule does not apply.

Explain independent events and the multiplication rule.

Meaning

The outcome of one event does not change the probability of the other.

Multiplication rule

P(A and B) = P(A) × P(B).

A coin is tossed and a die rolled. Find P(head and number less than 3).

½ × ⅓ = ⅙.

A ball is drawn from 2 red and 3 white, replaced, and drawn again. Find P(both white).

(3/5) × (3/5) = 9/25.

Use a tree diagram for two tosses of a coin.

Draw the first stage.

Two branches, H and T, each with probability ½.

Draw the second stage.

From each branch, two more branches H and T, each ½, giving HH, HT, TH, TT.

Find P(exactly one head).

HT + TH = ¼ + ¼ = ½.

Find P(at least one head).

1 – P(TT) = 1 – ¼ = ¾.

Multiple Choice Questions (MCQs)

Probability of a certain event is: (A) 0 (B) ½ (C) 1 (D) 2
Correct answer: (C) 1. Certain = 1.

Which cannot be a probability? (A) 0.7 (B) ⅗ (C) 1.2 (D) 0
Correct answer: (C) 1.2. Greater than 1.

P(head) on a fair coin: (A) 0 (B) ½ (C) 1 (D) ¼
Correct answer: (B) ½. Even chance.

P(6) on a fair die: (A) ⅙ (B) ⅓ (C) ½ (D) 1
Correct answer: (A) ⅙. One of six.

If P(A) = 0.4, P(not A) = (A) 0.4 (B) 0.6 (C) 1.4 (D) 0
Correct answer: (B) 0.6. 1 – 0.4.

A bag has 2 red, 3 blue, 5 green. P(blue) = (A) 0.2 (B) 0.3 (C) 0.5 (D) 0.6
Correct answer: (B) 0.3. 3/10.

30 heads in 80 tosses gives experimental P(head): (A) 0.375 (B) 0.5 (C) 0.3 (D) 0.8
Correct answer: (A) 0.375. 30/80.

Expected number of sixes in 300 rolls: (A) 30 (B) 50 (C) 60 (D) 100
Correct answer: (B) 50. 300 × ⅙.

Mutually exclusive events have P(A and B) = (A) 1 (B) P(A)P(B) (C) 0 (D) ½
Correct answer: (C) 0. Cannot occur together.

For mutually exclusive events P(A or B) = (A) P(A)P(B) (B) P(A) + P(B) (C) P(A) – P(B) (D) 1
Correct answer: (B) P(A) + P(B). Addition rule.

For independent events P(A and B) = (A) P(A) + P(B) (B) P(A) × P(B) (C) 0 (D) 1
Correct answer: (B) P(A) × P(B). Multiplication rule.

Two coins tossed: number of outcomes: (A) 2 (B) 3 (C) 4 (D) 6
Correct answer: (C) 4. HH, HT, TH, TT.

Two dice: P(sum = 7): (A) 1/36 (B) ⅙ (C) 7/36 (D) ½
Correct answer: (B) ⅙. 6/36.

Two dice: P(sum = 12): (A) 1/36 (B) 1/12 (C) ⅙ (D) 1/18
Correct answer: (A) 1/36. Only (6, 6).

On a tree diagram, probabilities along a path are: (A) Added (B) Multiplied (C) Subtracted (D) Divided
Correct answer: (B) Multiplied. AND rule.

Quick Revision Summary

- $0 \le P \le 1$: impossible 0, even chance ½, certain 1.
- Theoretical P = favourable ÷ total; P(not A) = 1 – P(A).
- Experimental P = successes ÷ trials; expected frequency = P × trials.
- Mutually exclusive: P(A or B) = P(A) + P(B).
- Independent: P(A and B) = P(A) × P(B).
- Tree diagrams: multiply along branches, add the paths.

Exam Tips

- Write the sample space first; count carefully.
- Leave answers as simplified fractions unless decimals are asked for.

- Use 1 – P for 'at least one' questions.
- Check that branch probabilities from each point add up to 1.
- Decide first whether events are 'or' (add) or 'and' (multiply).